Differentialgeometrie und Analysis
Fachbereich Mathematik und Informatik
Philipps-Universität Marburg
27.03. - 30.3.2017

1. Marburger Arbeitsgemeinschaft Mathematik (MAM)

(Non)-existence of complex structures on S^6

(1) Almost complex structures on spheres

Using characteristic classes and Bott periodicity one can prove that the only spheres that may admit almost complex structures are S^2 and S^6.

References: [Kir47, Hop48, Fri78, MT91, Mur09, Hat09, Pos91]

Speaker: Maurizio Parton

Contact person in Marburg: Panagiotis Konstantis

Talks: 1-2

(2) S^6 as a nearly Kähler manifold

Nearly Kähler structures are one of the 16 classes of almost Hermitian structures discovered by Gray and Hervella. The sphere S^6 carries a nearly Kähler structure, whose almost complex structure can be described using octonions or spinors.

References: [Cal58, BFGK91, Fri06, Agr06, Mur09, FH15]

Speaker: Aleksandra Borówka

Contact persons in Marburg: Ilka Agricola, Stefan Vasilev

Talks: 1

(3) Orthogonal complex structures near the round S^6

An almost complex structure on a Riemannian manifold is *orthogonal* if it is an isometry on each tangent space. In [LeB87], LeBrun showed that an orthogonal almost complex structure on the round S^6 is never integrable. This results was generealized in [Gil] as follows: if g is a Riemannian metric on S^6 belonging to a certain neighbourhood of the round metric, then (S^6, g) does not admit an orthogonal complex structure.

References: [LeB87, Mus89, Sal96, BHL99, Wil16]

Speaker: Ana Cristina Ferreira, Boris Kruglikov

Contact person in Marburg: Oliver Goertsches

Talks: 2-3

(4) Dolbeault cohomology and Frölicher spectral sequence

Dolbeault cohomology and the Frölicher spectral sequence are standard topics in complex geometry. Dolbeault cohomology combines the study of differential forms on a manifold with the presence of a complex structure. The Frölicher spectral sequence measures how far is Dolbeault cohomology from de Rham cohomology.

References: [GH78, Voi02, Huy05]

Speaker: Benedict Meinke

Contact persons in Marburg: Giovanni Bazzoni, Sönke Rollenske

Talks: 1

(5) Hodge numbers of a hypothetical complex structure on S^6

The Hodge numbers of a complex manifold are computed by Dolbeault cohomology. Under the hypothesis that S^6 has a complex structures, one can compute the corresponding Hodge numbers. In particular, although S^6 is simply connected, the Hodge number $h^{0,1}$ is non-zero for a hypothetical complex structure.

References: [Gra97, Uga00]

Speaker: Daniele Angella

Contact person in Marburg: Giovanni Bazzoni

Talks: 1-2

(6) Algebraic dimension and automorphism group of a hypothetical complex structure on S^6

The algebraic dimension of a complex manifold is the transcendence degree of the field of meromorphic functions over $\mathbb{C}$. Since the Euler characteristic of S^6 is non-zero, it follows that the algebraic dimension of a hypothetical complex S^6 should be zero. Moreover, a hypothetical complex S^6 is not almost homogeneous, i.e. the group of holomorphic automorphisms does not have an open orbit.

References: [Hop48, CDP98, HKP00]

Speaker: Christian Lehn, Caren Schinko

Contact persons in Marburg: Giovanni Bazzoni, Sönke Rollenske

Talks: 3

(7) The exceptional Lie group G_2

The exceptional Lie group G_2 and its properties will be discussed.

References: [Cal58, Sal03, Agr08]

Speaker: Cristina Draper Fontanals

Contact person in Marburg: Ilka Agricola

Talks: 1

(8) Chern's contribution

In 2003, S.-s. Chern began a study of almost complex structures on S^6, with the idea of exploiting the special properties of its well-known almost complex structure invariant under the exceptional group G_2. He proved a significant identity that solves the question for an interesting class of almost complex structures.

References: [Bry14]

Speaker: Aleksy Tralle, Markus Upmeier

Contact person in Marburg: Thomas Friedrich

Talks: 2

(9) Further approaches to the (non)-existence

In the last part we discuss different approaches to a positive, or negative, solution to the Hopf problem.

References: [Ati16, Ete15a, Ete15b, Ete15c]

Speaker: Ben Anthes, Tim Kirschner

Contact persons in Marburg: Ilka Agricola, Thomas Friedrich

References

[Agr06] Ilka Agricola. The Srní lectures on non-integrable geometries with torsion. *Arch. Math. (Brno)*, 42(suppl.):5–84, 2006.

[Agr08] Ilka Agricola. Old and new on the exceptional group G_2. *Notices Amer. Math. Soc.*, 55(8):922–929, 2008.

[Ati16] Michael Atiyah. The non-existent complex 6-sphere. *arXiv:1610.09366*, 2016.

[BFGK91] Helga Baum, Thomas Friedrich, Ralf Grunewald, and Ines Kath. *Twistors and Killing spinors on Riemannian manifolds*, volume 124. B. G. Teubner Verlagsgesellschaft mbH, Stuttgart, 1991.

[BHL99] Gil Bor and Luis Hernández-Lamoneda. The canonical bundle of a Hermitian manifold. *Bol. Soc. Mat. Mexicana (3)*, 5(1):187–198, 1999.

[Bry14] Robert Bryant. S.-s. Chern's study of almost complex structures on the six-sphere. *arXiv:1405.3405*, 2014.

[Cal58] Eugenio Calabi. Construction and properties of some 6-dimensional almost complex manifolds. *Trans. Amer. Math. Soc.*, 87:407–438, 1958.

[CDP98] Frédéric Campana, Jean-Pierre Demailly, and Thomas Peternell. The algebraic dimension of compact complex threefolds with vanishing second Betti number. *Compositio Math.*, 112(1):77–91, 1998.

[Ete15a] Gábor Etesi. Complex structure on the six dimensional sphere from a spontaneous symmetry breaking. *J. Math. Phys.*, 56(4):043508, 21, 2015.

[Ete15b] Gábor Etesi. Erratum: "Complex structure on the six dimensional sphere from a spontaneous symmetry breaking". *J. Math. Phys.*, 56(9):099901, 1, 2015.

[Ete15c] Gábor Etesi. Explicit construction of the complex structure on the six dimensional sphere. *arXiv:1509.02300*, 2015.

[FH15] Lorenzo Foscolo and Mark Haskins. New G_2 holonomy cones and exotic nearly Kähler structures on the 6-sphere and the product of a pair of 3-spheres. *arXiv:1501.07838*, 2015.

[Fri78] Thomas Friedrich. *Vorlesungen über K-Theorie*. B. G. Teubner-Verlagsgesellschaft, Leipzig, 1978. With English, Russian and French summaries, Teubner-Texte zur Mathematik.

[Fri06] Thomas Friedrich. Nearly Kähler and nearly parallel G_2-structures on spheres. *Arch. Math. (Brno)*, 42(suppl.):241–243, 2006.

[GH78] Phillip Griffiths and Joseph Harris. *Principles of algebraic geometry*. Wiley-Interscience [John Wiley & Sons], New York, 1978. Pure and Applied Mathematics.

[Gil]

[Gra97] Alfred Gray. A property of a hypothetical complex structure on the six sphere. *Boll. Un. Mat. Ital. B (7)*, 11(2, suppl.):251–255, 1997.

[Hat09] Allen Hatcher. Vector Bundles and K-Theory. `http://www.math.cornell.edu/~hatcher/VBKT/VB.pdf`, 2009.

[HKP00] Alan T. Huckleberry, Stefan Kebekus, and Thomas Peternell. Group actions on S^6 and complex structures on $\mathbf{P}_3$. *Duke Math. J.*, 102(1):101–124, 2000.

[Hop48] H. Hopf. Zur Topologie der komplexen Mannigfaltigkeiten. In *Studies and Essays Presented to R. Courant on his 60th Birthday, January 8, 1948*, pages 167–185. Interscience Publishers, Inc., New York, 1948.

[Huy05] Daniel Huybrechts. *Complex geometry*. Universitext. Springer-Verlag, Berlin, 2005. An introduction.

[Kir47] Adrian Kirchoff. Sur l'existence des certains champs tensoriels sur les sphères à n dimensions. *C. R. Acad. Sci. Paris*, 225:1258–1260, 1947.

[LeB87] Claude LeBrun. Orthogonal complex structures on S^6. *Proc. Amer. Math. Soc.*, 101(1):136–138, 1987.

[MT91] Mamoru Mimura and Hirosi Toda. *Topology of Lie Groups, I and II*, volume 91. Translations of Mathematical Monographs, 1991.

[Mur09] Stefan Murygin. Das Hopfproblem: Über die Existenz komplexer Strukturen auf der sechs-dimensionalen Sphäre. Diplomarbeit, HU Berlin, 2009.

[Mus89] Emilio Musso. On the twistor space of the six-sphere. *Bull. Austral. Math. Soc.*, 39(1):119–127, 1989.

[Pos91] Michail Postnikov. *Leçons de géométrie - Variétés différentiables*. Traduit du russe, Éditions Mir Moscou, 1991.

[Sal96] Simon Salamon. Orthogonal complex structures. In *Differential geometry and applications (Brno, 1995)*, pages 103–117. Masaryk Univ., Brno, 1996.

[Sal03] Simon Salamon. A tour of exceptional geometry. *Milan J. Math.*, 71:59–94, 2003.

[Uga00] Luis Ugarte. Hodge numbers of a hypothetical complex structure on the six sphere. *Geom. Dedicata*, 81(1-3):173–179, 2000.

[Voi02] Claire Voisin. *Hodge theory and complex algebraic geometry. I*, volume 76 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2002. Translated from the French original by Leila Schneps.

[Wil16] Scott O. Wilson. Results concerning almost complex structures on the six-sphere. *arXiv:1610.09620*, 2016.