Proof of the Riemann Hypothesis utilizing the theory of Alternative Facts

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January 24, 2017

Abstract

Conway's powerful theory of Alternative Facts can render many difficult problems tractable. Here we demonstrate the power of AF to prove the Riemann Hypothesis, one of the most important unsolved problems in mathematics. We further suggest applications of AF to other challenging unsolved problems such as the zero-equals-one conjecture (which is also true) and the side-counting problem of the circle. We determine the circle to have exactly 11 unique sides.

1 Problem statement

The Riemann Hypothesis has remained one of the most important unsolved problems in modern mathematics. RH connects discrete mathematics to analytic mathematics and its truth or falsity has implications for hundreds of other important unsolved questions from number theory to topology to algebra. RH has until now resisted solution under mathematical theories based on standard facts.

The Riemann Hypothesis states that the zeros of Riemann ζ function occur only at the negative even integers and points in the complex plane with real part equal to 1/2. We'll apply Conway's Alternative Fact framework to prove this conjecture to be true.

2 Conway's Alternative Facts

Conway's Theory of Alternative Facts (AF) is a recent break-through in mathematical logic allowing for propositions to be either True or False depending on context or motive of the agent positing the fact. While the traditional logic, first introduced by Aristotle and formalized in the 19th century, associates a unique Boolean value to any proposition, AF allows for more fluid associations. For any proposition, P, we associate a *context* C and a *motive* M and a decision function which maps the triplet (P, C, M) to the space of Booleans, $f : (P, C, M) \rightarrow \{0, 1\}$.

The breakthrough with Alternative Facts is the concept of the *context* and *motive*. A context is a maximal set of propositons, axiomatic or otherwise, which can simultaneously be considered true. While the context as such does not offer anything beyond the standard concept of formal systems, they become considerably more powerful when we combine them with *motives*. A motive, intuitively speaking, describe, the purpose of the propositons and the reasoning steps that can be applied to them. Formally, they consist of constraints on the lengths of logical chains of reasoning within the system. They separate logical chains of reasoning into *pro-motive* chains and *counter-motive* chains. Pro-motive chains are the logical chains of reasoning that lead from propositions explicitly included in the context to the desired logical conclusion. Counter-motive chains lead from the same propositons to the negation of the desired conclusion. Any chain of reasoning has a length consisting of the number of logical steps taken. Within a motive, we define limits to each class of chain. Typically, the length limit of counter-motive chains is chosen to be smaller than for pro-motive chains.

A system of Alternative Facts is considered consistent when any attempt to establish a contradiction within the system requires more logical steps than allowed by the length limit of counter-motive chains. This is particularly useful in using AFs to arrive at a desired outcome when reasoning time and space is limited such as often occurs in situations such as class room lectures, print publication and television interviews.

3 Proof of the Riemann Hypothesis using AF

We first posit the proposition which is the standard statement of the RH. In this case, we can take the context to be a private one consisting solely of this proposition and the statement that the ζ function is equal to the function $\theta(z) = z - 1/2$ at all points in the positive real half-plane and analytically continued outside of that plane. The motive, M, is taken simply as the desire to prove RH and we choose a limit of ∞ and 0 for pro-motive and counter-motive chains respectively.

This function clearly has a unique critical point at $z = 1/2$ therefore proving the RH within this context and under this motive. By extending this context to include all other mathematical statements, except those contradicting RH, we prove RH in the most general context in which it is true. Maximally general contexts are referred to as *huge*.

We note however that this does not preclude the existence of other contexts and motives in which the hypothesis is false. We leave as an open problem the question of whether such a context exists and whether it is huge.

4 Difficulties involved in standard proofs

Here we point out the critical part played by AF in this proof. In a traditional fact framework, positing the equality of $\zeta(z)$ and our $\theta(z)$ runs into several problems. By utilizing the mechanism of Selberg's trace formula or alternatively Mangold's explicit formula one can show that every prime number is equal to either 4 or 8 in contradiction with the known factorizations of 4 and 8 and the known impossibility of factoring 3, 5 and other larger known prime numbers. Attempts to find a way around these difficulties utilizing psuedo-Riemannian manifolds, p-adic circular abrigation and quasi-formal induction have all proved dead ends.

These failures to prove RH by equating ζ with the θ-function all fail for the same reason. In each case, it leads to the existence of propositions and their negation both of which can be proven true. The existence of propositions that are both true and false is impossible in the standard Aristotelian logic. And so, progress on the problem was blocked until Conways's breakthrough in introducing Alternative Fact ontologies.

5 The Zero-Equals-One conjecture

Mathematicians have long wondered whether a consistent formal system can be constructed in which the normal rules of arithmetic hold and in addition $0 = 1$. The chief difficulty in these model concerns the construction of higher numbers when adding 1 must result in the same value as adding 0, which through the systems axioms, must leave the number unchanged. If the original number must be 0 or 1 (which by construction are identical anyway), forming numbers not equal to these is challenging. We will now show how Conways AF theory can provide a way forward.

We choose as our context the standard Peano Axoims for the set of natural numbers and the identity 0=1. Our motive is the construction of higher numbers and, as above, we limit the number of counter-motive steps to a small number. A length of 1 suffices. We then construct the higher numbers by adding the number 1. Since by the axioms, adding 1 results in a different number, we achieve our result.

Now adding 1 to a number must be the same as adding 0 to a number and, through our axioms, adding 0 to a number results in the same number. This is therefore a contradiction. However this chain required utilizing two base axioms and the application of 2 logical steps which lies outside of our stated motive.

6 Circles have 11 sides

The proof of this statement is straight-forward. Context is that circles have 11 sides and that all other properties of circles from Euclidean geometry hold. This, therefore is a huge context. Motive is simply to avoid the existence of circles with a number of sides unequal to 11. We will prove this using the technique of *reductio ad absurdum*.

Assume that a circle of radius R, with $R > 1$, has N sides. Since N is a finite number we can compare the number of sides to the known number of 11 using the side-counting procedure. This requires N logical steps and so as long as we allow for pro-motive logical chain lengths of at least N, we can construct such a chain for any N. If N is unequal to 11 for any N (other than 11 of course), we have arrived at a contradiction within out motive which proves that circles must have exactly 11 sides.

Finally, our motive must limit counter-motive chains lengths to 10 which precludes the existence of logical arguments based on side-counting that would conflict with other known results of plane geometry that depend on circles not having a finite number of sides. This completes the proof. The case of $R \leq 1$ is nearly identical and so we leave this to the reader.

7 Conclusion

Using Conways's theory of Alternative Facts we prove the famous Riemann Hypothesis. Our proof is constructive, providing an explicit context and motive in which the theorem holds. In addition, we show how the power of AF can be used to solve other challenging unsolved problems. We expect the methods employed herein to generalize to many other unsolved problems and even some proven theorems that can now also be proven otherwise.

Alternative facts have been incorrectly compared to falsehoods or out-right lies. We hope in this paper to have corrected that misperception. Moreover, some have argued that utilizing alternative facts allows for anything to be declared true making argumentation itself pointless. But this is not the case. As long as two parties agree on a context and motive, any argumentation becomes just as binding as an argument with standard facts. Difficulties may indeed arise when context and motive are different. However, even in that case, the only limitation is the inability of the two parties to reach a unique set of conclusions. It is our experience that arguments between parties with different context and motives and expectations of agreement rarely occur in practice.

References

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