Deforming G_2 Conifolds

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Joint with Spiro Karigiannis (Waterloo)

Motivation

Conifolds

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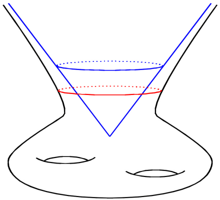

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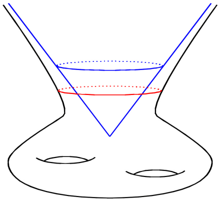

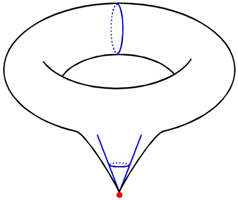

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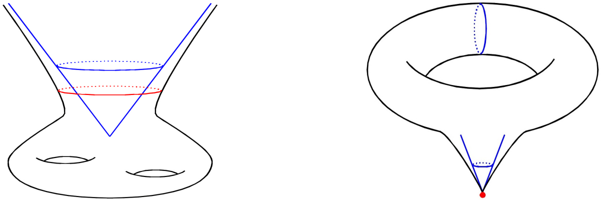

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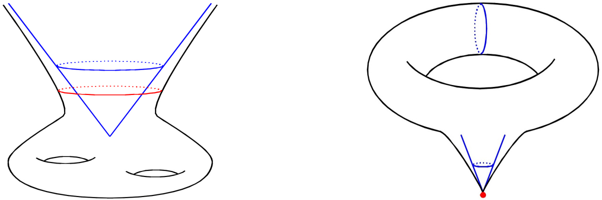

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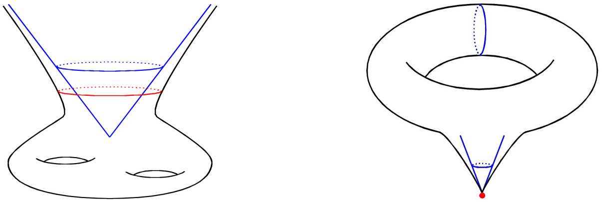

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- New examples/local uniqueness of holonomy G_2 metrics

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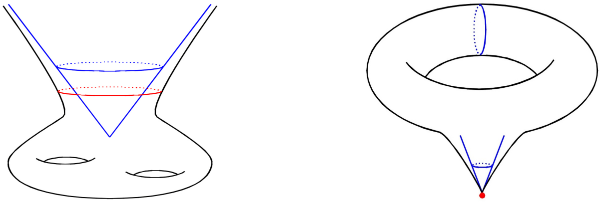

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- Moduli space of compact G_2 manifolds
- New examples/local uniqueness of holonomy G_2 metrics
- Relevance to M-Theory

Outline

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- **Definitions and examples**

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- **Open problems**

G_2 structures

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M^7 connected, oriented and spin

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- (Joyce–Karigiannis) Potential method for constructing CS holonomy G_2 manifolds, $\Sigma = \mathbb{CP}^3$

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- $b^3_{cs}(M) + \dim \operatorname{Im}\left(H^3(M) \to H^3(\Sigma)\right) + \sum_{\lambda \in (-3,\nu)} m_\Sigma(\lambda)$ if $\nu \in (-3, -5/2)$

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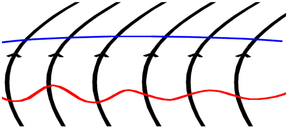

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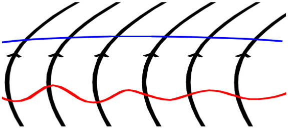

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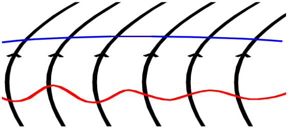

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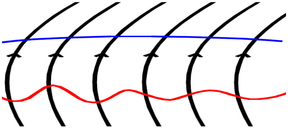

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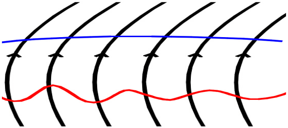

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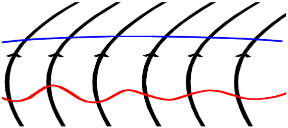

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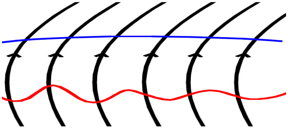

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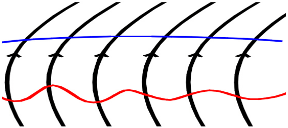

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