SODEs of the adjoint orbits of the closed fundamental real forms of $G$. The Kummer--Kleinschmidt--Kronheimer--Manivel--Matsumoto orbit example is given as the orbit that corresponds to the adjoint orbit of the 2-dimensional $G$-module $\mathfrak{g}/\mathfrak{g}^\mathfrak{g}/\mathfrak{g}$. The problem of identifying the adjoint orbit of the 2-dimensional $G$-module $\mathfrak{g}/\mathfrak{g}^\mathfrak{g}/\mathfrak{g}$ is an open problem in the theory of Lie algebras.

Riemannian geometry in its dimensions

Let $G = O(2)$ be the Riemannian manifold of dimension $2$. Then, we denote the Lie group $\text{SO}(2)$ by $G$.

Definition 1. The Kummer--Kleinschmidt--Kronheimer--Matsumoto orbit example is given as the orbit that corresponds to the adjoint orbit of the 2-dimensional $G$-module $\mathfrak{g}/\mathfrak{g}^\mathfrak{g}/\mathfrak{g}$.

In the above diagram, the 2-dimensional $G$-module $\mathfrak{g}/\mathfrak{g}^\mathfrak{g}/\mathfrak{g}$ is decomposed into two distinct subspaces.

A Klein correspondence

Proposition 1. Let $G = O(2)$ and let $\pi : G \to \text{SO}(2)$ be the projection map. Then, $\pi$ is a Klein correspondence.

Mumford polytopes

Proposition 2. The Mumford polytopes $P_{\text{Mum}}$ of $G = O(2)$ are polytopes given by $P_{\text{Mum}} = \text{conv}(\{e\})$.

Cone polytopes

Proposition 3. The cone polytopes $P_{\text{cone}}$ of $G = O(2)$ are polytopes given by $P_{\text{cone}} = \text{conv}(\{e\})$.

Applications

Theorem 4. The Klein correspondence of the maximum $T$ of $\text{SO}(2)$ is a Klein correspondence.

Example 1. Consider the Klein correspondence of the maximum $T$ of $\text{SO}(2)$.

Example 2. Consider the Klein correspondence of the maximum $T$ of $\text{SO}(2)$.