Generalized Cardinal B-Splines: Stability, Linear Independence, and Appropriate Scaling Matrices

Stephan Dahlke, Vera Latour, and Michael Neeb
Institut für Geometrie
und Praktische Mathematik
RWTH Aachen
Templergraben 55
52056 Aachen
Germany

Abstract

Generalized cardinal B-splines are defined as convolution products of characteristic functions of self-affine lattice tiles with respect to a given integer scaling matrix. By construction, these generalized splines are refinable functions with respect to the scaling matrix and therefore they can be used to define a multiresolution analysis and to construct a wavelet basis. In this paper, we study the stability and linear independence properties of the integer translates of these generalized spline functions. Moreover, we give a characterization of the scaling matrices to which the construction of the generalized spline functions can be applied.

Key Words: Generalized B-splines, stability, linear independence, self-affine tilings, multiresolution analysis, wavelets.

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