Besov Regularity for Elliptic Boundary Value Problems

Stephan Dahlke *
Institut für Geometrie
und Praktische Mathematik
RWTH Aachen
Templergraben 55
52056 Aachen
Germany

Ronald A. DeVore †
Department of Mathematics
University of South Carolina
Columbia, S.C. 29208
U.S.A.

Abstract

This paper studies the regularity of solutions to boundary value problems for the Laplace operator on Lipschitz domains Ω in $\mathbb{R}^d$ and its relationship with adaptive and other nonlinear methods for approximating these solutions. The smoothness spaces which determine the efficiency of such nonlinear approximation in $L_p(\Omega)$ are the Besov spaces $B^\alpha_\tau(L_\tau(\Omega))$, $\tau := (\alpha/d + 1/p)^{-1}$. Thus, the regularity of the solution in this scale of Besov spaces is investigated with the aim of determining the largest α for which the solution is in $B^\alpha_\tau(L_\tau(\Omega))$. The regularity theorems given in this paper build upon the recent results of Jerison and Kenig [10]. The proof of the regularity theorem uses characterizations of Besov spaces by wavelet expansions.

Key Words: Besov spaces, elliptic boundary value problems, potential theory, adaptive methods, nonlinear approximation, wavelets.

AMS Subject Classifications: Primary 35B65, secondary 31B10, 41A46, 46E35, 65N30.

*The work of this author has been supported by Deutsche Forschungsgemeinschaft (Da 360/1–1).

†The work of this author has been supported by the Office of Naval Research Contract N0014-91-J1343.