Stable Multiscale Bases and Local Error Estimation for Elliptic Problems

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Abstract

This paper is concerned with the analysis of adaptive multiscale techniques for the solution of a wide class of elliptic operator equations covering, in principle,

singular integral as well as partial differential operators. The central objective is to derive reliable and efficient a-posteriori error estimators for Galerkin schemes which are based on stable multiscale bases. It is shown that the locality of corresponding multiresolution processes combined with certain norm equivalences involving weighted sequence norms of wavelet coefficients leads to adaptive space refinement strategies which are guaranteed to converge in a wide range of cases, again including operators of negative order.

Key Words: Stable multiscale bases, norm equivalences, elliptic operator equations, Galerkin schemes, a-posteriori error estimators, convergence of adaptive schemes.