Wavelets: Construction Principles and Applications to the Numerical Treatment of Operator Equations

Stephan Dahlke *
Institut für Geometrie
und Praktische Mathematik
RWTH Aachen
Templergraben 55
52056 Aachen
Germany

Abstract

This thesis is concerned with a new and rapidly expanding field of mathematics, that is, with wavelet analysis. The present study consists of two major parts. The first part is devoted to the construction of wavelets. We are primarily interested in the construction of multivariate wavelet bases for general scalings. We derive a systematic approach which is based on the theory of self–affine lattice tilings and which applies to a large class of nontrivial scaling matrices. Moreover, we study in detail the properties of the resulting wavelet bases. Especially, we derive conditions which guarantee that the wavelets are sufficiently smooth and have stable integer translates. We also construct wavelets which are adapted to a given pseudo–differential operator in the sense that functions on different refinement levels are orthogonal with respect to the bilinear form induced by the operator. The second major part is concerned with the applications of wavelets to the numerical treatment of operator equations. Especially, we want to clarify how wavelet analysis can be used to design and to analyze adaptive numerical methods. In general, the efficiency of an adaptive scheme depends on the regularity of the exact solution in a specific scale of Besov spaces. We use wavelet analysis to investigate the Besov regularity for a large class of elliptic problems, especially for second order partial differential equations. The proofs of the regularity theorems use the fact that smoothness spaces such as Besov spaces can be characterized by certain weighted sequence norms of the coefficients of wavelet expansions. Furthermore, we present a suitable adaptive strategy. We derive a posteriori error estimators based on wavelet expansions which are both reliable and efficient. These error estimators lead to adaptive space refinement strategies

*This work has been supported by the Deutsche Forschungsgemeinschaft (Da 360/1–1).

which are guaranteed to converge in a wide range of cases, including operators of negative order.