# Nonlinear Approximation and Adaptive Techniques for Solving Elliptic Operator Equations

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**Abstract**

This survey article is concerned with two basic approximation concepts and their interrelation with the numerical solution of elliptic operator equations, namely nonlinear and adaptive approximation. On one hand, for nonlinear approximation based on wavelet expansions, the best possible approximation rate which a function can have for a given number of degrees of freedom is characterized in terms of its regularity in a certain scale of Besov spaces. Therefore, after demonstrating the gain of nonlinear approximation over linear approximation measured in a Sobolev scale, we review some recent results on the Sobolev and Besov regularity of solutions to elliptic boundary value problems. On the other hand, nonlinear approximation requires information that is generally not available in practice. Instead one has to resort to the concept of adaptive approximation. We briefly summarize some recent results on wavelet based adaptive schemes for elliptic operator equations. In contrast to more conventional approaches one can show that these schemes converge without prior assumptions on the solution such as the saturation property. One central objective of this paper is to contribute to interrelating nonlinear approximation and adaptive methods in the context of elliptic operator equations.

**Key Words:** Nonlinear approximation, adaptive methods, elliptic equations, wavelets, characterization of function spaces.

**AMS Subject Classifications:** 35B65, 41A46, 46E35, 65N12, 65N30, 65N55.