Adaptive Wavelet Methods for Saddle Point Problems*

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Abstract

Recently, adaptive wavelet strategies for symmetric, positive definite operators have been introduced that were proven to converge. This paper is devoted to the generalization to saddle point problems which are also symmetric, but indefinite. Firstly, we derive explicit criteria for adaptively refined wavelet spaces in order to fulfill the *Ladyshenskaja–Babuška–Brezzi (LBB)* condition and to be fully equilibrated. Then, we investigate a posteriori error estimates and generalize the known adaptive wavelet strategy to saddle point problems. The convergence of this strategy for elliptic operators essentially relies on the positive definite character of the operator. As an alternative, we introduce an adaptive variant of Uzawa's algorithm and prove its convergence. Finally, we detail our results for two concrete examples of saddle point problems, namely the mixed formulation of the Stokes problem and second order elliptic boundary value problems where the boundary conditions are appended by Lagrange multipliers.

Keywords: Adaptive schemes, a posteriori error estimates, multiscale methods, wavelets, saddle point problems, Uzawa's algorithm, Stokes problem, Lagrange multipliers.

AMS Subject Classifications: 42C15, 65N55.

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