Smooth Refinable Functions and Wavelets Obtained by Convolution Products

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Abstract

This paper is concerned with the construction of smooth refinable functions relative to a large class of expanding scaling matrices. Characteristic functions of certain self-similar tiles related to a given scaling matrix are the simplest examples of such refinable functions. It is known that a sufficiently high convolution power of such characteristic function produces eventually refinable functions of arbitrary high regularity. The objective of this paper is to quantify the number of convolutions needed to achieve continuous differentiability. This turns out to be possible when using convolutions of possibly different judiciously chosen tiles associated with the same scaling matrix. An essential ingredient of our analysis is the concept of stationary subdivision schemes which allows us to derive explicit estimates for the smoothness of the resulting convolution products. Once a regular refinable function is obtained we briefly point out how to construct a corresponding multiresolution analysis and wavelets.

Key Words: Wavelets, expanding scaling matrices, multiresolution analysis, self–similar tilings, stationary subdivision.

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