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DFG

DFG-Projekt: “Multivariate Wavelet Analysis: Constructions, Specific Applications, and Data Structures” SPP 1114

General Guideline:

Do the superiorities of general scalings really count when it comes to practical applications, or are they wasted by an overhead of technical difficulties?

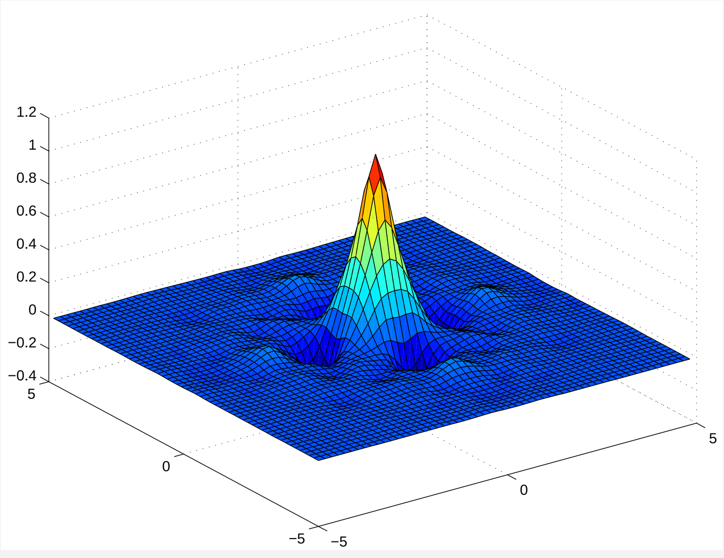
Construction of Wavelets for General Scalings

These wavelet bases are constructed by means of a scaling function satisfying a two–scale relation

$$\phi(x) = \sum_{k \in \mathbb{Z}^d} a_k \phi(Mx - k),$$

where $M \in \mathbb{Z}^{d \times d}$ is an expanding integer scaling matrix. For various applications, interpolating scaling functions, $\phi(k) = \delta_{0,k}$, are needed. Our aim is to construct ϕ as smooth and localized as possible. To obtain stable algorithms, it is furthermore essential to identify suitable biorthogonal bases

$$\langle \phi(\cdot), \tilde{\phi}(\cdot - k) \rangle = \delta_{0,k}.$$



Smooth dual of an interpolating scaling function with Hölder regularity 1.952

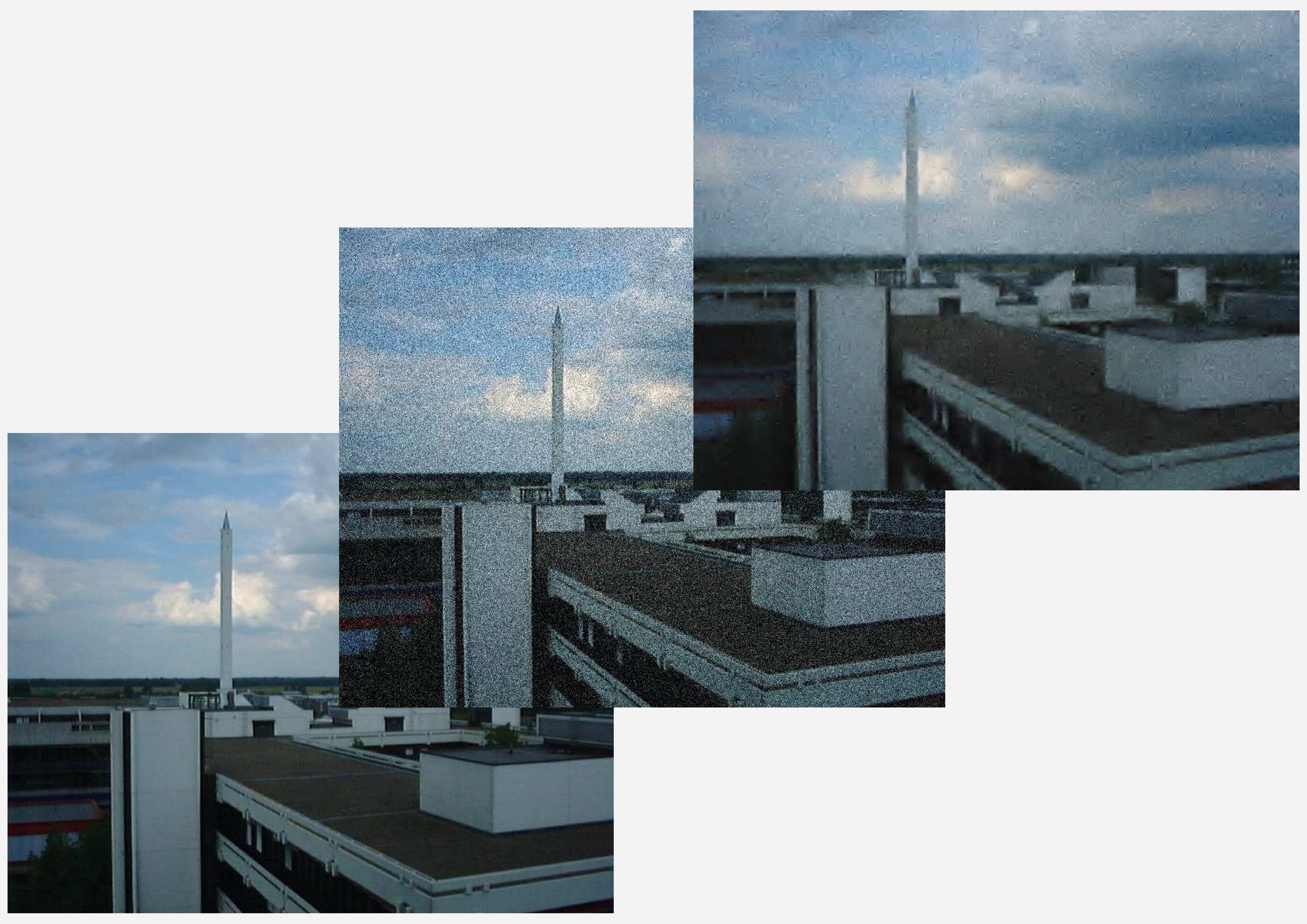
The construction of smooth and localized dual functions is nontrivial, especially in higher dimensions. Later on, we shall also investigate how the flexibilities of multiwavelets can be exploited.

Applications in Image Processing

Based on these new wavelet bases, we intend to develop efficient denoising algorithms. This is usually performed by a thresholding strategy, i.e. by applying the shrinkage operator

$$s_\varepsilon(t) = \begin{cases} t - \varepsilon & : t > \varepsilon \\ 0 & : |t| \le \varepsilon \\ t + \varepsilon & : t < -\varepsilon \end{cases}$$

to the wavelet coefficients. Then the choice of the thresholding parameter ε is the crucial step. For i.i.d. Gaussian noise, optimal shrinkage parameters have been derived by DeVore et.al. by solving specific variational problems. One of our aims is to generalize these results to other types of noise such as colored or nonstationary noise.



Especially, we want to find out if the use of general scalings or multiwavelets is advantageous. The investigations will be accompanied by the development and incorporation of suitable data structures. These problems are closely related with the efficiency of best N-term approximation schemes. We intend to characterize the power of these schemes for general scalings and the multiwavelet setting.

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http://www.math.uni-bremen.de/zetem/DFG-Schwerpunkt/