Universität Bremen

Reconstruction of Reflectivity Densities by Frame Techniques

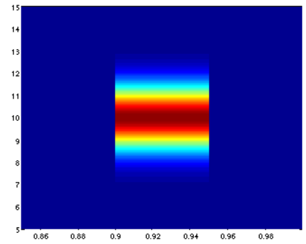
Radar Problem:

The basic radar problem asks to gain information about an object by analyzing waves reflected from it. The distance between object and source at time $t = 0$ is denoted be R. The emitted signal is denoted by $h(t)$, then the *wideband model* for the received echo $f(t)$ is given by

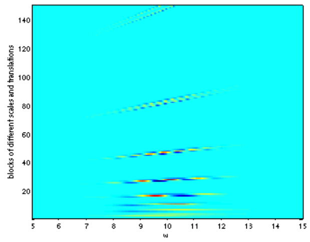
$$f(t) \;=\; \sqrt{|s|}\; h(s(t-\tau)),$$

where the *Doppler scale factor* s and the delay τ are obtained from the speed of light c, the object velocity v and the distance R by $s = (c-v)/(c+v),\; \tau = (2R)/(c-v)$.
In the presence of many objects the total echo is modeled by a *reflectivity density* $D(\tau, 1/s)$. Then the echo is given by

$$f(t) = \int_{\mathbf{R}} \int_{\mathbf{R}\setminus\{0\}} D(\tau,s)|s|^{-1/2} h\left(\frac{t-\tau}{s}\right) \frac{ds d\tau}{s^2}.$$



Reflectivity density



Observed echoes

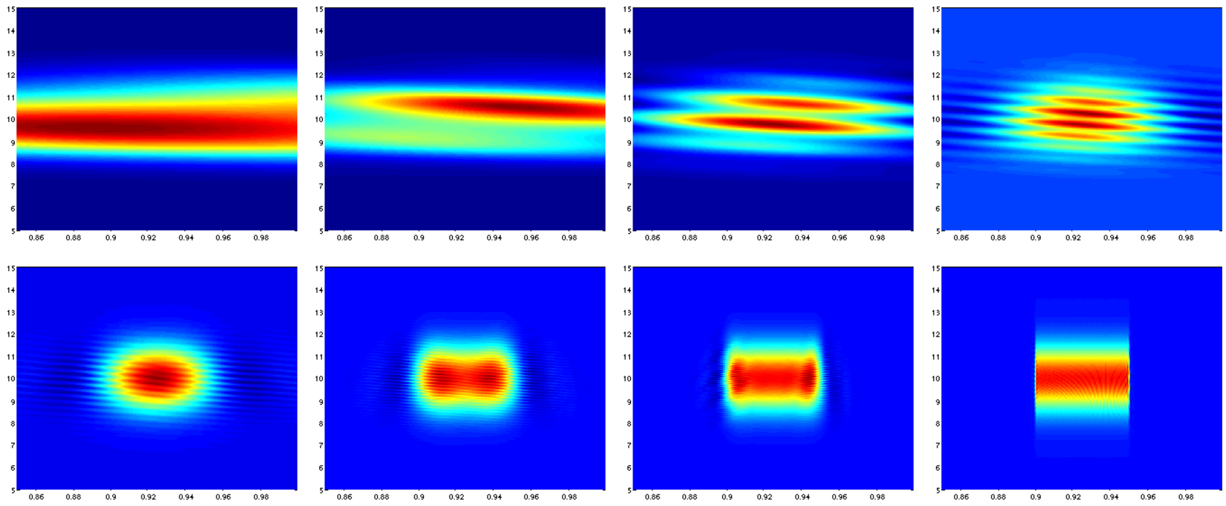
Density Reconstruction:

Consequently, the task is to reconstruct the density $D(\tau,s)$ from the received echo. We suggest not to transmit just *one* signal but a *frame* $\{h_m\}_{m\in\mathbf{Z}}$ of signals. Then every function F has an expansion

$$F = \sum_{m\in\mathbf{Z}} \langle F, h^m\rangle h_m = \sum_{m\in\mathbf{Z}} \langle F, h_m\rangle h^m, \quad h^m := T^{-1}h_m, \quad T(F) := \sum_{m\in\mathbf{Z}} \langle F, h_m\rangle h_m.$$

In this setting, the basic reconstruction formula reads as follows:

$$4\pi^2 D(\tau,s) = \sum_{m\in\mathbf{Z}} \int_{-\infty}^{0} -\frac{1}{i}\widehat{f_m'}(\omega)\widehat{h^m(\frac{\cdot}{s})}(\omega)|s|^{1/2}e^{i\tau\omega}d\omega + \sum_{m\in\mathbf{Z}} \int_{0}^{\infty} \frac{1}{i}\widehat{f_m'}(\omega)\widehat{h^m(\frac{\cdot}{s})}(\omega)|s|^{1/2}e^{i\tau\omega}d\omega.$$



Reconstruction of the reflectivity density using the frame algorithm (Cohen–Daubechies–Feauveau biorthogonal wavelet frame)

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