Universität Bremen

# Reconstruction of Reflectivity Densities by Frame Techniques II

## Multivariate Model:

We want to reconstruct *multivariate* densities for echoes generated by the reflection of a signal $h(x,t)$. Suppose that the object is moving with velocity $v = (v_x, v_y)$ in the $(x,y)$–plane and that the signal $h(x,t)$ is transmitted. Then the echo produced by a *single* object is given by

$$e\left(\tau, x_0 + v_x\left(\frac{(1+s_0)\tau - \tau_0}{2s_0}\right)\right) = h\left(\frac{\tau - \tau_0}{s_0}, x_0 + v_x\left(\frac{(1+s_0)\tau - \tau_0}{2s_0}\right)\right).$$

The echo produced by a *multivariate reflectivity density* $D(\tau_0, s_0, z_0, \sigma_0)$ can be described as

$$e(\tau, x) = \int_{\mathbf{R}}\int_{\mathbf{R}}\int_{\mathbf{R}}\int_{\mathbf{R}} D(\tau_0, s_0, z_0, \sigma_0)\delta(x - (z_0 + \sigma_0\tau))h(\frac{\tau - \tau_0}{s_0}, z_0 + \sigma_0\tau)d\tau_0 ds_0 d\sigma_0 dz_0,$$

where $\sigma_0 = v_x(1+s_0)/(2s_0)$ and $z_0 = x_0 - (v_x\tau_0)/(2s_0)$.

The nullspace is characterized by a three-dimensional set of conditions. In order to characterize the complement of the nullspace, i.e., the reconstructable reflectivity distributions, we therefore need to eliminate one of the four variables of $D \in L_2(\mathbf{R}^4)$. This can be achieved by reducing the dependency of $D$ on $\sigma_0$ by fixing the mean velocity as follows: we define

$$\mathcal{D}(\tau_0, s_0, x, t) := \int_{\mathbf{R}} D(\tau_0, s_0, x - \sigma_0 t, \sigma_0)d\sigma_0.$$

This leads to

$$e(\tau, x) \sim h_n(x)\int_{\mathbf{R}}\int_{\mathbf{R}} \mathcal{D}(\tau_0, s_0, x, v_0/c)h_m\left(\frac{\tau - \tau_0}{s_0}\right)d\tau_0 ds_0\ ,$$

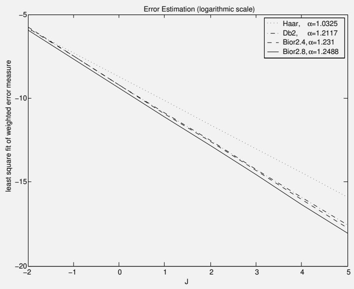
which can be reconstructed by the frame algorithm.

## Error Estimates:

Suppose that for some fixed $\alpha$ a Jackson type estimate of the form

$$\|g - \frac{1}{(2\pi)}\sum_{m\in I_J}\langle g, \hat{h}_m\rangle \hat{h^m}\|^2_{L_2(\mathbf{R})} \le C\ 2^{-2J\alpha}\ |g|^2_{H^\alpha(\mathbf{R})}$$

holds. Then following error estimate in a weighted $L_2$-norm is valid:




Least square fit of evaluated error in the logarithmic scale

$$\int_{\mathbf{R}}\int_{\mathbf{R}}\left|D(\tau, s) - \frac{1}{(2\pi)^2}\sum_{m\in I_J}\int_{\mathbf{R}}\hat{f}_m(\omega)\hat{h}_m(s\omega)|\omega||s|^{3/2}e^{i\tau\omega}d\omega\right|^2 d\tau\frac{ds}{|s|^3} \le C2^{-2J\alpha}\int_{\mathbf{R}}|\omega||\widetilde{D}(\omega,\cdot)|^2_{H^\alpha}d\omega\ ,$$

where $\widetilde{D}(\omega,\sigma) = \widehat{D(\cdot, \sigma/\omega)}(\omega)|\omega|^{1/2}|\sigma|^{-3/2}$. It is well–known that such a Jackson type estimate holds for the Cohen–Daubechies–Feauveau biorthogonal wavelet system.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