

Workshop Tensor categories, Hopf algebras and quantum groups - Marburg, January 22-26, 2018

	Monday 22	Tuesday 23	Wednesday 24	Thursday 25	Friday 26
09:00 - 10:00	Lectures 1A (HS IV) 2A (HS I)	Lectures 1B (HS IV) 3A (HS I)	Lectures 3B (HS IV) 2B (HS I)	Lectures 1C (HS IV) 2C (HS I)	Lecture 3C (HS IV)
10:00 - 10:30	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
10:30 - 11:10	Christoph Schweigert	Nicolás Andruskiewitsch	Julien Bichon	Ulrich Krähmer	Réamonn Ó Buachalla
11:10 - 11:50	Kenichi Shimizu	Christian Kassel	Ralf Meyer	Cristian Vay	Leandro Guarneri
11:50 - 12:30	Peter Schauenburg	Giovanna Carnovale	Xin Fang	Julian Külshammer	Lukas Woike
12:30 - 14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00 - 14:40	Martin Mombelli	Azat Gainutdinov	Talk by Joachim Cuntz Hörsaal der Mathematik 17:00 hs. Gießen	Sergey Neshveyev	Departure
14:40 - 15:20	Henry Tucker	Simon Lentner		Thomas Timmermann	
15:20 - 15:50	Coffee break	Coffee break		Coffee break	
15:50 - 16:30	Sebastian Posur	Michael Cuntz		Kenny de Commer	
16:30 - 17:10	Tanmay Deshpande	Ehud Meir		Stefan Jung	
17:10 - 17:50	Lorant Szegedy	Matthias Klupsch		individual discussions	

Courses:

Christian Voigt (1A,1B,1C)

Title: *An introduction to complex semisimple quantum groups*

Abstract: Complex semisimple quantum groups are deformations of complex semisimple Lie groups obtained using the Drinfeld double construction. A basic example is the quantum Lorentz group $SL_q(2, \mathbb{C})$ introduced by Podleś-Woronowicz, which is the Drinfeld double of $SU_q(2)$. These quantum groups appear naturally in a variety of contexts, and the aim of these talks is to discuss their representation theory and links with the study of tensor categories and subfactors, operator K-theory and the Baum-Connes conjecture.

Ingo Runkel (2A,2B,2C)

Title: *Finite tensor categories*

Abstract: Finite tensor categories can be thought of as a generalisation of the categories of representations of finite-dimensional Hopf algebras. In particular, they only possess a finite number of simple objects, each of which has a projective cover of finite length. In these lectures I will review some general tools and results for these categories, as well as some examples.

Leandro Vendramin (3A,3B,3C)

Title: *Nichols algebras over non-abelian groups*

Abstract: Nichols algebras appear in several branches of mathematics going from Hopf algebras and quantum groups, to Schubert calculus and conformal field theories. In this talk we review the some results related to Nichols algebras, we discuss some classification theorems and we recall open problems and conjectures.

Talks:

Nicolás Andruskiewitsch

Title: *On Nichols algebras over basic Hopf algebras*

Abstract: TBA.

Julien Bichon

Title: Homological properties of quantum permutation algebras.

Abstract: I will discuss some homological properties of the universal quantum permutation algebras, and in particular I will present the recent result that they are Calabi-Yau of dimension 3. Talk based on joint work with U. Franz and Malte Gerhold.

Réamonn Ó Buachalla

Title: *Schubert Calculus for the Quantum Grassmannians*

Abstract: We present the Heckenberger-Kolb calculus for the quantum Grassmannians as the Nichols algebra of a Yetter-Drinfeld braiding. We use this presentation to construct a covariant noncommutative Kahler structure for the calculus, and apply the resulting Hodge theoretic tools to the study of the cohomology of the calculus. Using a noncommutative generalisation of the classical character map, we show that the cohomology groups have classical dimension. More surprisingly, in low dimensional examples we show that the cohomology ring is isomorphic to the classical Schubert ring of the Grassmannians. Time permitting, we will discuss a newly established generalisation of Kodaira vanishing to the setting of noncommutative Kahler structures, and discuss its application to the construction of spectral triples for quantum projective space.

Giovanna Carnovale

Title: *Nichols algebras over finite simple groups of Lie type*

Abstract: I will discuss how the difference in the structure of conjugacy classes in finite groups of Lie type relates to the existence of a finite-dimensional Nichols algebra attached to a given class. Joint work with N. Andruskiewitsch and G. García.

Michael Cuntz

Title: *What are the Weyl groupoids of Nichols algebras of diagonal type?*

Abstract: To classify finite dimensional Nichols algebras of diagonal type, Heckenberger introduced the so called arithmetic root systems and their symmetry structures, the Weyl groupoids. These tools turned out to be very successful and produced many remarkable classifications. The case when the Weyl groupoid does not have a finite root system was however much less studied. In this talk we propose a way to extend the old results for larger classes of Nichols algebras of diagonal type by allowing root systems defining certain given convex cones.

Kenny De Commer

Title: *Three categorical pictures for quantum symmetric spaces*

Abstract: Using Tannaka-Krein methods, a duality can be constructed between actions of a compact quantum group on the one hand, and module C^* -categories over its representation category on the other. In this talk, we will construct three module C^* -categories for the q -deformed representation category of a compact semisimple Lie group G , starting from a compact symmetric space G/K for G . The first construction is based on the theory of cyclotomic KZ-equations developed by B. Enriquez. The second construction uses the notion of quantum symmetric pair as developed by G. Letzter. The third construction uses the notion of twisted Heisenberg algebra. In all cases, we show that the module C^* -category is twist-braided - this is due to B. Enriquez in the first case, S.

Kolb in the second case, and closely related to work of J. Donin, P. Kulish and A. Mudrov in the third case. We formulate a conjecture concerning equivalence of these twist-braided module C^* -categories, and prove the equivalence in the simplest case of quantum $SU(2)$. This is joint work with S. Neshveyev, L. Tuset and M. Yamashita.

Tanmay Deshpande

Title: *Centers of bimodule categories and induction-restriction functors*

Abstract: Modular categories, multifusion categories and their module categories play an important role in the theory of character sheaves on algebraic groups. Lusztig defined character sheaves on (possibly disconnected) reductive groups using certain induction and restriction functors. In this talk, I will describe a toy categorical analogue of Lusztig's theory in the setting of a bimodule category over a multifusion category and the center of the bimodule category.

Xin Fang

Title: *From quantum groups to tropical flag varieties*

Abstract: Commutation relations between two quantum PBW root vectors are studied by Levendorskii and Soibelman in the early 90's, while the monomials appearing in a commutation relation are hard to determine. In this talk I will explain a polyhedral approach to this problem, showing how it is related to the tight monomial cone of Lusztig, as well as to the tropical complete flag varieties. This talk is based on an ongoing joint work with Ghislain Fourier and Markus Reineke.

Azat Gainutdinov

Title: *Modified trace is a symmetrised integral*

Abstract: A modified trace for a pivotal Hopf algebra H is a family of linear functionals on endomorphism spaces of projective H -modules which has cyclicity property and satisfies the so-called partial trace condition defined by the pivotal structure. We show that giving a modified trace is equivalent to Calabi-Yau structure on the category of projective H -modules and this structure is compatible with the duality. The modified traces provide a meaningful generalization of the categorical trace to non-semisimple categories and allow to construct interesting topological invariants. Our main theorem says that a modified trace is determined by a symmetric linear form constructed from an integral on the Hopf algebra. More precisely, we prove that for any finite dimensional unimodular pivotal Hopf algebra shifting with the pivotal element defines an isomorphism between the space of right integrals, which is known to be 1-dimensional, and the space of modified traces. This is a joint work with Anna Beliakova and Christian Blanchet.

Leandro Guarnieri

Title: *Skew Braces and the Yang-Baxter equation*

Abstract: Braces were introduced by Rump as a generalization of radical rings to study non-degenerate involutive set-theoretic solutions of the Yang-Baxter equation. We present skew braces as a generalization of classical braces to study not necessarily involutive non-degenerate set-theoretical solutions. We give some properties and examples of skew braces and link them to other subjects. We present an algorithm to enumerate and construct classical and skew braces of small size (up to isomorphism). This is joint work with Leandro Vendramin. See arXiv:1511.03171.

Stefan Jung

Title: *Partition Quantum Spaces*

Abstract: T. Banica and R. Speicher used (categories of) partitions to define special classes of compact matrix quantum groups, so called easy quantum groups. I will talk about a similar machinery to construct so called partition quantum spaces. Starting with a (suitable) set of partitions and considering both the associated easy quantum group G and the partition quantum space X , I will discuss in how far these two objects fit together in the sense of matrix-vector actions: We will see that we always have actions $G \curvearrowright X$ and $X \curvearrowleft G$. We further try to recover G as the so called quantum symmetry group of the quantum space X . This is joint work with Moritz Weber and based on the arXiv preprint 1801.06376.

Christian Kassel

Title: *Distinguishing comodule algebras using polynomial identities*

Abstract: I will talk about a project with Eli Aljadeff whose aim is to distinguish certain classes of comodule algebras with the help of adequate polynomial identities, thus extending similar results for group graded algebras. In my talk I will define polynomial identities for comodule algebras and give some examples where they can be used to distinguish comodule algebras.

Matthias Klupsch

Title: *On finite-dimensional semisimple Hopf algebras of a special type and matched pairs of groups*

Abstract: In this talk, we shall consider finite-dimensional semisimple Hopf algebras whose simple modules are one-dimensional except for one isomorphism class. By a construction of Masuoka, one can use matched pairs of groups to construct examples of such Hopf algebras. We shall present a complete classification of these matched pairs.

Ulrich Krähmer

Title: *How homogeneous is a quantum homogeneous space?*

Abstract: In this talk I will report on work with Angela Tabiri and with Manuel Martins in which the coordinate ring of a singular plane curve is embedded as a quantum homogeneous space into a suitable Hopf algebra.

Julian Külshammer

Title: *Existence and uniqueness of exact Borel subalgebras*

Abstract: Quasi-hereditary algebras and their infinite analogues, highest weight categories, appear frequently in many areas of representation theory. In joint work with S. Koenig and S. Ovsienko, we showed that every quasi-hereditary algebra can up to Morita equivalence be obtained as the dual of a coring object in the tensor category of bimodules over a directed algebra, the exact Borel subalgebra. Under an additional assumption, the exact Borel subalgebra as well as the coring object are in fact unique up to isomorphism. In this talk I will review these results. This additionally includes joint work with V. Miemietz as well as G. Jasso.

Simon Lentner

Title: *Nichols algebras acting on conformal field theories*

Abstract: I will present a recent result that any quantum group (and more generally any Nichols algebra of diagonal type) acts on an infinite-dimensional vector space via so-called screening operators, as has been conjectured for some time. I will not assume knowledge of theoretical physics. The key step of the proof is that certain analytical functions (generalized Selberg integrals) have zeroes whenever the Nichols algebra has a relation. Further conjectures related an associated modular tensor category to the representations of the quantum group. This demonstrated that general Nichols algebras of diagonal type and their representation theory appear naturally in complex analysis and via this in theoretical physics.

Ehud Meir

Title: *Hopf algebras, monoidal categories and geometric invariant theory*

Abstract: In this talk I will describe a study of Hopf algebras by tools from symmetric monoidal categories and geometric invariant theory. I will explain how the study of finite dimensional semisimple Hopf algebras can be reduced into studying scalar invariants, and explain the connections of these scalars to some well known invariants (such as the Reshetikhin Turaev invariants of 3-manifolds and Frobenius-Schur indicators). I will also explain how one can receive some new finiteness results for Hopf algebras by using symmetric monoidal categories. More precisely, I will explain why every finite dimensional semisimple Hopf algebra admits at most finitely many Hopf orders over a given number ring.

Ralf Meyer

Title: *Braided multiplicative unitaries as regular objects*

Abstract: I explain the theory of regular objects in tensor categories. This clarifies the passage between braided multiplicative unitaries and multiplicative unitaries with projection. The latter describe quantum group analogues of semidirect products. The braided multiplicative unitary and its semidirect product multiplicative unitary have the same Hilbert space representations. The multiplicative unitaries associated to two regular objects for the same tensor category are equivalent and hence generate isomorphic C^* -quantum groups. In particular, a C^* -quantum

group is determined uniquely by its tensor category of representations on Hilbert space, and any functor between representation categories that does not change the underlying Hilbert spaces comes from a morphism of C^* -quantum groups.

Martín Mombelli

Title: *Group actions on 2-categories*

Abstract: I will show the definition of a discrete group action on a 2-category and some examples coming from the theory of tensor categories.

Sergey Neshveyev

Title: *Dynamical characterisation of categorical Morita equivalence*

Abstract: Given two compact quantum groups, a bimodule category over their representation categories, together with a generating object, corresponds to a unital C^* -algebra endowed with commuting actions of these quantum groups. We show that such a bimodule category is invertible if and only if the actions are free, with finite dimensional fixed point algebras, which are in duality as Frobenius algebras in an appropriate sense. This extends the well-known characterization of monoidal equivalence in terms of bi-Hopf-Galois objects. (Joint work with Makoto Yamashita.)

Sebastian Posur

Title: *Computable tensor categories*

Abstract: We explain the concepts of constructive category theory and computable tensor categories together with their implementations in our software project Cap - Categories, algorithms, programming. As an application of Cap's framework we discuss a constructive skeletal model of the tensor category of finite dimensional representations of a finite group over a splitting field.

Peter Schauenburg

Title: *Numerical invariants of group-theoretical fusion categories*

Abstract: We consider some numerical invariants of (modular) fusion categories, notably the Frobenius-Schur indicators defined for any spherical fusion category, and the modular data of modular fusion categories. In general, these invariants are of considerable interest for the structure theory and classification of such categories, but it is by no means easy to calculate them or understand their general behavior. In the case of group-theoretical fusion categories, explicit formulas exist in terms of the groups and cohomology data defining the categories; this still does not imply that the results of these formulas are easy to calculate in specific examples or that the explicit results are easy to understand conceptually. On the one hand, we will report on a rather brutal computer-based attempt to calculate invariants and put them to use for classification purposes: We classify pointed fusion categories associated to small size groups up to Morita equivalence (or, equivalently, twisted Drinfeld doubles of finite groups up to equivalence); we will try to give an overview of the results, and explain the techniques involved and the point where we get stuck both because of the size of the problem and because of the need for more tools. On the other hand, we will report on a class of (counter-)examples we found in our computer based exploits (although they do not need computer help to be explained after all): There exist distinct modular categories that share the same modular data.

Kenichi Shimizu

Title: *Hochschild cohomology of finite tensor categories*

Abstract: Since the Hochschild cohomology is a Morita invariant, the Hochschild cohomology is well-defined for finite abelian categories. The Hochschild cohomology of a Hopf algebra can be computed by the cohomology of its adjoint representation. In my talk, I will explain how to extend this result to finite tensor categories and exact module categories over them. As an application, we obtain a projective action of the modular group $SL_2(\mathbb{Z})$ on the Hochschild cohomology of a "non-semisimple" modular tensor category (in the sense of Lyubashenko). A key ingredient is the "adjoint algebra" of a finite tensor category, which is a generalization of the adjoint representation of a Hopf algebra defined in terms of the Drinfeld center.

Christoph Schweigert

Title: *Eilenberg-Watts calculus for finite categories and a bimodule Radford S^4 theorem*

Abstract: We obtain Morita invariant versions of Eilenberg-Watts type theorems, relating Deligne products of finite linear categories to categories of left exact as well as of right exact functors. This makes it possible to switch between different functor categories as well as Deligne products, which is often very convenient. For instance, we can show that applying the equivalence from left exact to right exact functors to the identity functor, regarded as a left exact functor, gives a Nakayama functor. The equivalences of categories we exhibit are compatible with the structure of module categories over finite tensor categories. This leads to a generalization of Radford's S^4 -theorem to bimodule categories. We also explain the relation of our construction to relative Serre functors on module categories that are constructed via inner Hom functors.

Lorant Szegedy

Title: *TQFT on r -spin surfaces and the Arf invariant*

Abstract: We present a state-sum construction of TQFTs on r -spin surfaces which uses a combinatorial model of r -spin structures. We give an example of such a TQFT which computes the Arf invariant for r even. We use the combinatorial model and this TQFT to calculate diffeomorphism classes of r -spin bordisms.

Thomas Timmermann

Title: *Partial actions and the quantum Bernoulli shift of a discrete quantum group*

Abstract: Partial actions of a group arise as restrictions of global actions to non-invariant subspaces or ideals, and are closely related to actions of the transformation groupoid of the Bernoulli shift of the group. We discuss the passage from groups to discrete quantum groups and present a quantum Bernoulli shift that is closely related to a partial Hopf algebroid constructed by Alves, Batista and Vercruyssen in the algebraic context.

Henry Tucker

Title: *TBA*

Abstract: TBA

Cristian Vay

Title: *On the representation theory of a quantum group attached to the Fomin-Kirillov algebra FK_3*

Abstract: Let D be the Drinfeld double of the bosonization of a finite-dimensional Nichols algebra over a finite-dimensional semisimple Hopf algebra H . Then, D admits a triangular decomposition which allows us to study the representation theory of D by imitating the methods of Lie Theory. For instance, we can construct the simple modules as the head of the Verma modules. Moreover, the category of graded D -modules is highest-weight where the simple of the Drinfeld double of H play the role of the weights. This fact makes the computation in concrete examples much more complicated than in Lie Theory, where the product of two weights is again a weight. Instead, if H is not an abelian group, the product of two weights is a sum of weights.

In this talk we will explain how to handle this problem. We will describe the simple modules, their extensions and their tensor products for the Drinfeld double attached to the Fomin-Kirillov algebra FK_3 and the symmetric group S_3 . This is based on joint works with B. Pogorelsky.

Lukas Woike

Title: *A TFT perspective on orbifoldization*

Abstract: We provide a geometric construction which assigns in a functorial way to an equivariant topological field theory a non-equivariant topological field theory. This construction unifies and generalizes several known algebraic notions of orbifoldization and effectively implements an integration over all twisted sectors of the equivariant theory. Our orbifold construction generalizes to a push operation along arbitrary group morphisms.