

$$\begin{array}{ccc}
\mathcal{P}|1 \overset{\mathbb{R}}{\underset{0}{\triangleleft}} & \ni & \mathfrak{L}_\lambda P_\lambda \\
\downarrow & & \\
\tilde{\mathcal{O}}|1 & \ni & \int^{\mathbb{R}} \mathfrak{R} \mathfrak{L}_\lambda = \int_{dP}^{\mathbb{R}} \mathfrak{L}
\end{array}$$

$$\mathfrak{L} \in \mathbb{R} \triangleleft_{\alpha|\beta}$$

\mathcal{A} part von $\alpha|\beta$

$$\mathcal{A} \ni A = \ell_A | r_A$$

$$|A| = r_A - \ell_A$$

$$|\mathcal{A}| = \max_{A \in \mathcal{A}} |A| = \max_{A \in \mathcal{A}} \underline{r_A - \ell_A}$$

$$P_A := P_{r_A} - P_{\ell_A} \in \mathcal{P}|1 \text{ proj-valued length}$$

$$\underline{\Sigma}_{\mathcal{A}} \mathfrak{L} = \sum_{A \ni A} \overset{A}{\wedge} \mathfrak{L} P_A \in \tilde{\mathcal{O}}|1 \ni \bar{\Sigma}_{\mathcal{A}} \mathfrak{L} = \sum_{A \ni A} \overset{A}{\Upsilon} \mathfrak{L} P_A$$

$$\underline{\Sigma}_{\mathcal{A}} \mathfrak{L} \leq \bar{\Sigma}_{\mathcal{A}} \mathfrak{L} \text{ in } \tilde{\mathcal{O}}|1$$

$$P_\lambda \text{ isoton} \Rightarrow P_A = P_{r_A} - P_{\ell_A} \geq 0 \Rightarrow \bar{\Sigma}_{\mathcal{A}} \mathfrak{L} - \underline{\Sigma}_{\mathcal{A}} \mathfrak{L} = \sum_{A \ni A} \underbrace{\overset{A}{\Upsilon} \mathfrak{L} - \overset{A}{\wedge} \mathfrak{L}}_{\geq 0} P_A \geq 0$$

$$\overline{\bar{\Sigma}_{\mathcal{A}} \mathfrak{L} - \underline{\Sigma}_{\mathcal{A}} \mathfrak{L}} \leq \max_{A \in \mathcal{A}} \overset{A}{\Upsilon} \mathfrak{L} - \overset{A}{\wedge} \mathfrak{L} = |\mathcal{A}|_{\mathfrak{L}}$$

$$0 \leq \bar{\Sigma}_{\mathcal{A}} \mathfrak{L} - \underline{\Sigma}_{\mathcal{A}} \mathfrak{L} = \sum_{A \ni A} \underbrace{\overset{A}{\Upsilon} \mathfrak{L} - \overset{A}{\wedge} \mathfrak{L}}_{\geq 0} P_A \leq |\mathcal{A}|_{\mathfrak{L}} \sum_{A \ni A} P_A = |\mathcal{A}|_{\mathfrak{L}} \overset{\text{proj}}{P}_{\alpha|\beta}$$

$$\Rightarrow \overline{\bar{\Sigma}_{\mathcal{A}} \mathfrak{L} - \underline{\Sigma}_{\mathcal{A}} \mathfrak{L}} \leq |\mathcal{A}|_{\mathfrak{L}} \overline{\overset{\leq 1}{P}_{\alpha|\beta}} \leq |\mathcal{A}|_{\mathfrak{L}}$$

$$\mathcal{A} \prec \mathcal{B} \Leftrightarrow \bigwedge_{B \in \mathcal{B}} \bigvee_{A \in \mathcal{A}} B \subset I \Leftrightarrow \mathcal{B} = \bigcup_{A \in \mathcal{A}} \frac{B \in \mathcal{B}}{B \subset I}$$

$$\alpha|\beta = \bigcup_{A \in \mathcal{A}} A = \bigcup_{B \in \mathcal{B}} B = \bigcup_{A \in \mathcal{A}} \bigcup_{B \in \mathcal{B} \ni B \subset I} B$$

$$A = \bigcup_{B \ni B \subset I} B \Rightarrow P_A = \sum_{B \ni B \subset I} P_B$$

$$\mathcal{A} \prec \mathcal{B} \Rightarrow \underline{\Sigma}_{\mathcal{A}} \mathfrak{L} \leq \underline{\Sigma}_{\mathcal{B}} \mathfrak{L} \leq \bar{\Sigma}_{\mathcal{B}} \mathfrak{L} \leq \bar{\Sigma}_{\mathcal{A}} \mathfrak{L}$$

$$\bar{\Sigma}_{\mathcal{B}} \mathfrak{L} = \sum_{B \ni B} \dot{Y}^B \mathfrak{L} P_B = \sum_{A \ni A} \sum_{B \ni B \subset I} \dot{Y}^B \mathfrak{L} P_B \leq \sum_{A \ni A} \dot{Y}^A \mathfrak{L} \sum_{B \ni B \subset I} P_B = \sum_{A \ni A} \dot{Y}^A P_A = \bar{\Sigma}_{\mathcal{A}} \mathfrak{L}$$

$$\bigvee_{\text{stop-limits}} \tilde{\mathfrak{U}} | \mathbb{1} \ni \int_* \mathfrak{L} := \lim_{\mathcal{A}}^s \underline{\Sigma}_{\mathcal{A}} \mathfrak{L} \leq \lim_{\mathcal{A}}^s \bar{\Sigma}_{\mathcal{A}} \mathfrak{L} = \int_*^* \mathfrak{L} \in \tilde{\mathfrak{U}} | \mathbb{1}$$

$$\mathfrak{L} \in \mathbb{R} \nabla_0^{\alpha|\beta} \Rightarrow \int_* \mathfrak{L} = \int_*^* \mathfrak{L} = \int_{dP_\lambda}^{\alpha|\beta} \mathfrak{L}_\lambda$$

$$\mathfrak{L} \text{ u-stet} \Rightarrow \bigwedge_{\varepsilon > 0} \bigvee_{\mathcal{A}} \bigwedge_{\mathcal{B} \prec \mathcal{A}} |\mathcal{B}|_{\mathfrak{L}} \leq \varepsilon$$

$$\Rightarrow 0 \leq \int_*^* \mathfrak{L} - \int_* \mathfrak{L} \leq \bar{\Sigma}_{\mathcal{B}} \mathfrak{L} - \underline{\Sigma}_{\mathcal{B}} \mathfrak{L} \xrightarrow{\text{pos}} \overline{\int_*^* \mathfrak{L} - \int_* \mathfrak{L}} \leq \overline{\bar{\Sigma}_{\mathcal{B}} \mathfrak{L} - \underline{\Sigma}_{\mathcal{B}} \mathfrak{L}} \leq |\mathcal{B}|_{\mathfrak{L}} \leq \varepsilon$$