

$$\begin{cases} m + 0 = m \\ m + n^+ = \overline{m + n}^+ \end{cases} \xRightarrow{\text{add}} \mathbb{N} \times \mathbb{N} \xrightarrow{+} \mathbb{N}$$

$$m + n \underset{\text{komm}}{=} n + m$$

$$m^+ + n = m + n^+$$

$$\begin{aligned} 0 = n: \quad m^+ + 0 &= m^+ = \overline{m + 0}^+ = m + 0^+ \\ 0 \leq n \curvearrowright n^+: \quad m^+ + n^+ &= \overline{m^+ + n^+}^+ \underset{\text{ind}}{=} \overline{m + n^+}^+ = m + n^{++} \end{aligned}$$

$$0 + n \underset{\text{Null}}{=} n$$

$$\begin{aligned} 0 = n: \quad 0 + 0 &= 0 \\ 0 \leq n \curvearrowright n^+: \quad 0 + n^+ &= \overline{0 + n^+}^+ \underset{\text{Vor}}{=} n^+ \end{aligned}$$

$$\begin{aligned} 0 = n: \quad m + 0 &= m \underset{\text{null}}{=} 0 + m \\ n \curvearrowright n^+: \quad m + n^+ &= \overline{m + n^+}^+ \underset{\text{ind}}{=} \overline{n + m}^+ \underset{\overline{n}}{=} n^+ + m^+ = n^+ + m \end{aligned}$$

$$\underline{a + b} + c \underset{\text{assoc}}{=} a + \underline{b + c}$$

$$m + q = n + q \xrightarrow[\text{cancel}]{\text{add}} m = n$$

$$M = \left\{ \begin{array}{l} q \in \mathbb{N} \\ m + q = n + q \curvearrowright m = n \end{array} \right\} = \left\{ \begin{array}{l} q \in \mathbb{N} \\ \mathbb{N} \xrightarrow[\text{inj}]{+q} \mathbb{N} \end{array} \right\} \cong 0$$

$$q \in M$$

$$m + \underline{q+1} = n + \underline{q+1} \Rightarrow \underline{m+1} + q = m + \underline{1+q} \stackrel{\text{Vor}}{=} n + \underline{1+q} = \underline{n+1} + q$$

$$\stackrel{\text{ind}}{\Rightarrow} m + 1 = n + 1 \stackrel{2N}{\Rightarrow} m = n \Rightarrow q + 1 \in M \stackrel{4N}{\Rightarrow} M = \mathbb{N}$$

$$\left\{ \begin{array}{l} \sum_0^k a_k = 0 \\ \sum_{n+1}^k a_k = \sum_m^n a_k + a_n \end{array} \right. \quad \stackrel{\text{finite}}{\Rightarrow} \text{sum} \quad \sum_m^n a_k = a_0 + \dots + a_{n-1}$$

$$\left\{ \begin{array}{l} 0 \cdot x = 0 \\ \underline{n+1}x = n \cdot x + x \end{array} \right. \quad \stackrel{\text{multiple}}{\Rightarrow} \quad n \cdot x = \sum_m^n x = x + \dots + x$$