

$$g = Edu\mathbf{x}du + 2Fdu\mathbf{x}dv + Gdv\mathbf{x}dv = \frac{1}{E} \left(Edu + \left(F + i\sqrt{EG - F^2} \right) dv \right) \left(Edu + \left(F - i\sqrt{EG - F^2} \right) dv \right)$$

$$\begin{aligned} \gamma \left(Edu + \left(F + i\sqrt{EG - F^2} \right) dv \right) &= dx + idy = \left(du \frac{\partial x}{\partial u} + dv \frac{\partial x}{\partial v} \right) + i \left(du \frac{\partial y}{\partial u} + dv \frac{\partial y}{\partial v} \right) \\ &= du \left(\frac{\partial x}{\partial u} + i \frac{\partial y}{\partial u} \right) + dv \left(\frac{\partial x}{\partial v} + i \frac{\partial y}{\partial v} \right) \end{aligned}$$

$$\begin{aligned} \bar{\gamma} \left(Edu + \left(F - i\sqrt{EG - F^2} \right) dv \right) &= dx - idy = \left(du \frac{\partial x}{\partial u} + dv \frac{\partial x}{\partial v} \right) - i \left(du \frac{\partial y}{\partial u} + dv \frac{\partial y}{\partial v} \right) \\ &= du \left(\frac{\partial x}{\partial u} - i \frac{\partial y}{\partial u} \right) + dv \left(\frac{\partial x}{\partial v} - i \frac{\partial y}{\partial v} \right) \end{aligned}$$

$$\gamma E = \frac{\partial x}{\partial u} + i \frac{\partial y}{\partial u}$$

$$\gamma \left(F + i\sqrt{EG - F^2} \right) = \frac{\partial x}{\partial v} + i \frac{\partial y}{\partial v}$$

$$\left(F + i\sqrt{EG - F^2} \right) \left(\frac{\partial x}{\partial u} + i \frac{\partial y}{\partial u} \right) = E \left(\frac{\partial x}{\partial v} + i \frac{\partial y}{\partial v} \right)$$

$$\begin{cases} F \frac{\partial x}{\partial u} - \sqrt{EG - F^2} \frac{\partial y}{\partial u} = E \frac{\partial x}{\partial v} \\ F \frac{\partial y}{\partial u} + \sqrt{EG - F^2} \frac{\partial x}{\partial u} = E \frac{\partial y}{\partial v} \end{cases} \Rightarrow \begin{cases} \frac{\partial y}{\partial u} = \frac{F \frac{\partial x}{\partial u} - E \frac{\partial x}{\partial v}}{\sqrt{EG - F^2}} \\ \frac{\partial x}{\partial u} = \frac{E \frac{\partial y}{\partial v} - F \frac{\partial y}{\partial u}}{\sqrt{EG - F^2}} \end{cases}$$

$$\left(F + i\sqrt{EG - F^2} \right) \left(F - i\sqrt{EG - F^2} \right) = EG$$

$$\left(F - i\sqrt{EG - F^2} \right) \left(\frac{\partial x}{\partial v} + i \frac{\partial y}{\partial v} \right) = G \left(\frac{\partial x}{\partial u} + i \frac{\partial y}{\partial u} \right)$$

$$\begin{cases} F \frac{\partial x}{\partial v} + \sqrt{EG - F^2} \frac{\partial y}{\partial v} = G \frac{\partial x}{\partial u} \\ F \frac{\partial y}{\partial v} - \sqrt{EG - F^2} \frac{\partial x}{\partial v} = G \frac{\partial y}{\partial u} \end{cases} \Rightarrow \begin{cases} \frac{\partial y}{\partial v} = \frac{G \frac{\partial x}{\partial u} - F \frac{\partial x}{\partial v}}{\sqrt{EG - F^2}} \\ \frac{\partial x}{\partial v} = \frac{F \frac{\partial y}{\partial v} - G \frac{\partial y}{\partial u}}{\sqrt{EG - F^2}} \end{cases}$$

$$0 = \frac{\partial}{\partial u} \frac{\partial x}{\partial v} - \frac{\partial}{\partial v} \frac{\partial x}{\partial u} = \frac{\partial}{\partial u} \frac{F \frac{\partial y}{\partial v} - G \frac{\partial y}{\partial u}}{\sqrt{EG - F^2}} - \frac{\partial}{\partial v} \frac{E \frac{\partial y}{\partial v} - F \frac{\partial y}{\partial u}}{\sqrt{EG - F^2}}$$

$$0 = \frac{\partial}{\partial v} \frac{\partial y}{\partial u} - \frac{\partial}{\partial u} \frac{\partial y}{\partial v} = \frac{\partial}{\partial v} \frac{F \frac{\partial x}{\partial u} - E \frac{\partial x}{\partial v}}{\sqrt{EG - F^2}} - \frac{\partial}{\partial u} \frac{G \frac{\partial x}{\partial u} - F \frac{\partial x}{\partial v}}{\sqrt{EG - F^2}}$$

$$\left(F + i\sqrt{EG - F^2} \right) \frac{\partial z}{\partial u} = E \frac{\partial z}{\partial v}$$

$$\left(F - i\sqrt{EG - F^2} \right) \frac{\partial z}{\partial v} = G \frac{\partial z}{\partial u}$$