

$$u_{\pm}^3 = q \pm \sqrt{q^2 - p^3}$$

$$\overline{u_+ u_-}^3 = u_+^3 u_-^3 = \underbrace{q + \sqrt{q^2 - p^3}}_q \underbrace{q - \sqrt{q^2 - p^3}}_q = q^2 - (q^2 - p^3) = p^3$$

$$\zeta_+ u_+ \zeta_- u_- = p$$

$$x = \zeta_+ u_+ + \zeta_- u_- \Rightarrow x^3 = 2q + 3px$$

$$\begin{aligned} \text{LHS} &= \overline{\zeta_+ u_+ + \zeta_- u_-}^3 = \overline{\zeta_+ u_+}^3 + \overline{\zeta_- u_-}^3 + 3 \overline{\zeta_+ u_+}^2 \zeta_- u_- + 3 \zeta_+ u_+ \overline{\zeta_- u_-}^2 \\ &= \underbrace{\zeta_+^3 u_+^3}_{=1} + \underbrace{\zeta_-^3 u_-^3}_{=1} + 3 \zeta_+^2 u_+^2 \zeta_- u_- + 3 \zeta_+ u_+ \zeta_-^2 u_-^2 \\ &= \underbrace{u_+^3 + u_-^3}_{=2q} + 3 \underbrace{\zeta_+ u_+ \zeta_- u_-}_{=p} \underbrace{\zeta_+ u_+ + \zeta_- u_-}_{=x} = \text{RHS} \end{aligned}$$