

$$t^{d/2} x e_y^{-t\Delta} = \sum_{j \geq 0} t^j x_j u_y$$

$$t^{d/2} \text{Tr} (e^{-t\Delta}) = t^{d/2} \int_{dg(x)}^M x e_x^{-t\Delta} \rightsquigarrow \int_{dg(x)}^M x_0 u_x = \text{Vol} (M)$$

$$\frac{t^{d/2} \text{Tr} (e^{-t\Delta}) - \text{Vol} (M)}{t} = t^{-1} \int_{dg(x)}^M \left( \sum_{j \geq 0} t^j x_j u_x - x_0 u_x \right)$$

$$= t^{-1} \int_{dg(x)}^M \sum_{j \geq 1} t^j x_j u_x = \int_{dg(x)}^M \sum_{j \geq 1} t^{j-1} x_j u_x \rightsquigarrow \int_{dg(x)}^M x_1 u_x = 6 \int \text{scal-curv}$$