

$$\mathbb{1} \subset \mathbb{1}_\alpha$$

$$\mathbb{1}_\alpha \xleftarrow[\text{proj}]{P_\alpha} \mathbb{1}$$

$$\bigwedge_{\alpha:\beta} \bigvee_{\gamma \supseteq \alpha:\beta} \mathbb{1}_\gamma \supset \mathbb{1}_\alpha \cup \mathbb{1}_\beta$$

$$\mathbb{1} \subset \mathbb{1}_\infty = \overline{\bigcup_\alpha \mathbb{1}_\alpha} \xleftarrow[\text{proj}]{P_\infty} \mathbb{1}$$

$$\Rightarrow P_\infty P_\alpha = P_\alpha$$

$$P_\infty \underset{\text{strong}}{\sim} P_\alpha : \bigwedge_{\mathbb{1} \in v} P_\infty \mathbb{1} \underset{\text{norm}}{\leq} P_\alpha \mathbb{1}$$

$$\text{OE } \mathbb{1} \in \mathbb{1}_\infty \Rightarrow \bigwedge_{k \in \mathbb{N}} \bigvee_{\alpha_k} \bigvee_{\mathbb{1} \in \mathbb{1}_{\alpha_k}} \overline{\mathbb{1} - \mathbb{1}} < \frac{1}{k}$$

$$\Rightarrow \bigwedge_{\beta \supseteq \alpha_k} \overline{P_\beta \mathbb{1} - \mathbb{1}} \underset{\text{best approx in } \mathbb{1}_\beta}{\leq} \overline{\underbrace{P_{\alpha_k} \mathbb{1}}_{\in \mathbb{1}_{\alpha_k} \subset \mathbb{1}_\beta} - \mathbb{1}} \underset{\text{best approx in } \mathbb{1}_{\alpha_k}}{\leq} \overline{\mathbb{1} - \mathbb{1}}_{\in \mathbb{1}_{\alpha_k}} < \frac{1}{k}$$

$$\Rightarrow P_\alpha \mathbb{1} \underset{\sim}{\sim} \mathbb{1}$$

$$\bigwedge_{\mathbb{1} \in \mathbb{1}_{\alpha_k}} \bigvee_{\alpha_k} P_\infty \mathbb{1} \leq P_{\alpha_k} \mathbb{1}$$

Martingal $\mathbb{1}_\alpha \in \mathbb{1}_\alpha$

$$\bigwedge_{\beta \supseteq \alpha} P_\beta \mathbb{1}_\beta = \mathbb{1}_\alpha$$

$$\bigwedge_{\beta \geq \alpha} \overline{\lrcorner_{\beta} - \lrcorner_{\alpha}} = \overline{\lrcorner_{\beta}} - \overline{\lrcorner_{\alpha}}$$

$$\begin{aligned} \lrcorner_{\alpha} \times \lrcorner_{\beta} &= \underbrace{P_{\iota} \lrcorner_{\alpha}} \times \lrcorner_{\beta} = \lrcorner_{\alpha} \times \underbrace{P_{\iota} \lrcorner_{\beta}} = \lrcorner_{\alpha} \times \lrcorner_{\alpha} \Rightarrow \lrcorner_{\alpha} \times \underbrace{\lrcorner_{\beta} - \lrcorner_{\alpha}} = 0 \\ \Rightarrow \overline{\lrcorner_{\beta} - \lrcorner_{\alpha}}^2 &= \underbrace{\lrcorner_{\beta} - \lrcorner_{\alpha}} \times \underbrace{\lrcorner_{\beta} - \lrcorner_{\alpha}} = \lrcorner_{\beta} \times \underbrace{\lrcorner_{\beta} - \lrcorner_{\alpha}} = \lrcorner_{\beta} \times \lrcorner_{\beta} - \lrcorner_{\beta} \times \lrcorner_{\alpha} = \lrcorner_{\beta} \times \lrcorner_{\beta} - \lrcorner_{\alpha} \times \lrcorner_{\alpha} \end{aligned}$$

$$\overline{\lrcorner_{\alpha}} \searrow$$

$$\overline{\lrcorner_{\alpha}} \text{ bes} \Rightarrow \lrcorner_{\alpha} \rightsquigarrow \lrcorner_{\infty} \in \mathbb{1}_{\infty}$$

$$P_{\iota} \lrcorner_{\infty} = \lrcorner_{\alpha}$$

$$\overline{\lrcorner_{\alpha}} \searrow \text{ bes} \Rightarrow \overline{\lrcorner_{\alpha}} \searrow c \Rightarrow \overline{\lrcorner_{\alpha}} \text{ Cau}$$