$$\text{Ⅎ} \sqsubset \text{Ⅎ}_\alpha$$

$$\text{Ⅎ}_\alpha \xleftarrow[\text{proj}]{P_\alpha} \text{Ⅎ}$$

$$\bigwedge_{\alpha:\beta} \bigvee_{\gamma \geqslant \alpha:\beta} \text{Ⅎ}_\gamma \supset \text{Ⅎ}_\alpha \cup \text{Ⅎ}_\beta$$

$$\text{Ⅎ} \sqsubset \text{Ⅎ}_\infty = \overline{\bigcup_\alpha \text{Ⅎ}_\alpha} \xleftarrow[\text{proj}]{P_\infty} \text{Ⅎ}$$

$$\Rightarrow P_\infty P_\alpha = P_\alpha$$

$$P_\infty \underset{\text{strong}}{\backsim} P_\alpha : \bigwedge_{\text{⅂} \in v} P_\infty \text{⅂} \underset{\text{norm}}{\Leftarrow} P_\iota \text{↿}$$

$$\text{OE } \text{⅂} \in \text{Ⅎ}_\infty \Rightarrow \bigwedge_{k \in \mathbb{N}} \bigvee_{\alpha_k} \bigvee_{\text{↿} \in \text{Ⅎ}_{\alpha_k}} \ulcorner \text{↿} - \text{⅂} \urcorner < \frac{1}{k}$$

$$\Rightarrow \bigwedge_{\beta \geqslant \alpha_k} \ulcorner P_\beta \text{⅂} - \text{⅂} \urcorner \underset{\text{best approx in } \text{Ⅎ}_\beta}{\leqslant} \ulcorner \underbrace{P_{\alpha_k} \text{⅂}}_{\in \text{Ⅎ}_{\alpha_k} \subset \text{Ⅎ}_\beta} - \text{⅂} \urcorner \underset{\text{best approx in } \text{Ⅎ}_{\alpha_k}}{\leqslant} \ulcorner \underset{\in \text{Ⅎ}_{\alpha_k}}{\text{↿}} - \text{⅂} \urcorner < \frac{1}{k}$$

$$\Rightarrow P_\iota \text{↿} \rightsquigarrow \text{⅂}$$

$$\bigwedge_{\text{⅂} \in \text{Ⅎ}} \bigvee_{\alpha_k \searrow} P_\infty \text{⅂} \Leftarrow P_{\alpha_k} \text{⅂}$$

$$\text{Martingal } \text{⅂}_\alpha \in \text{Ⅎ}_\alpha$$

$$\bigwedge_{\beta \geqslant \alpha} P_\iota \text{↿}_\beta = \text{⅂}_\alpha$$

$$\bigwedge_{\beta \geqslant \alpha} \overline{\rceil_\beta - \rceil_\alpha}^{\,2} = \overline{\rceil_\beta}^{\,2} - \overline{\rceil_\alpha}^{\,2}$$

$$\rceil_\alpha \star \rceil_\beta = \underbrace{P_\iota \upharpoonleft_\alpha} \star \rceil_\beta = \rceil_\alpha \star \underbrace{P_\iota \upharpoonleft_\beta} = \rceil_\alpha \star \rceil_\alpha \Rightarrow \rceil_\alpha \star \underbrace{\rceil_\beta - \rceil_\alpha} = 0$$

$$\Rightarrow \overline{\rceil_\beta - \rceil_\alpha}^{\,2} = \underbrace{\rceil_\beta - \rceil_\alpha} \star \underbrace{\rceil_\beta - \rceil_\alpha} = \rceil_\beta \star \underbrace{\rceil_\beta - \rceil_\alpha} = \rceil_\beta \star \rceil_\beta - \rceil_\beta \star \rceil_\alpha = \rceil_\beta \star \rceil_\beta - \rceil_\alpha \star \rceil_\alpha$$

$$\overline{\rceil_\alpha} \searrow$$

$$\overline{\rceil_\alpha} \text{ bes } \Rightarrow \rceil_\alpha \leadsto \rceil_\infty \in \perp_\infty$$

$$P_\iota \upharpoonleft_\infty = \rceil_\alpha$$

$$\overline{\rceil_\alpha} \searrow \text{ bes } \Rightarrow \overline{\rceil_\alpha} \searrow c \Rightarrow \overline{\rceil_\alpha} \text{ Cau}$$