

$\mathbb{R}^n \supset D$ bounded domain

∂D smooth

$$\Delta \begin{array}{c} \diagdown \\ D \\ \diagup \end{array} \begin{array}{c} \mathbb{R} \\ \leftarrow \mathcal{P} \\ \mathbb{R} \end{array} \begin{array}{c} \partial D \\ \leftarrow \\ \mathbb{R} \end{array}$$

$$z:s \in D \times \partial D \xrightarrow[\text{Poisson}]{\mathcal{P}} \mathbb{R} \ni {}^z \mathcal{P}_s$$

$$\Delta \mathcal{P} \mathcal{U} = 0$$

$${}^z \overline{\mathcal{P} \mathcal{U}} = \int_{\partial D} {}^z \mathcal{P}_s \cdot \mathcal{U} \, ds$$

$$\text{r-ball } D = \frac{z \in \mathbb{R}^n}{|z| < r}$$

$$|z| > r \Rightarrow {}_{\text{out}} {}^z \mathcal{P}_s = \sum_n^N (2n+1) \frac{r^{n+1}}{|z|^{n+1}} \underbrace{P_n(z \cdot s)}_{\text{Leg}}$$

$$|z| < r \Rightarrow {}_{\text{int}} {}^z \mathcal{P}_s = \sum_n^N (2n+1) \frac{|z|^{n+1}}{r^{n+1}} \underbrace{P_n(z \cdot s)}_{\text{Leg}}$$