

$$\bigvee_{\substack{\text{cpt Cartan} \\ \text{abel split}}} \mathfrak{h}^{\mathbb{R}} \supset \mathfrak{h}_{\mathbb{T}}^{\mathbb{R}} \supset \max_{\text{abel}} \mathfrak{h}_{\mathbb{T}}^{\mathbb{R}}$$

$$\mathfrak{h}^{\mathbb{R}} \supset \mathfrak{h}^{\mathbb{R}} = \mathfrak{h}_{\mathbb{T}}^{\mathbb{R}} \subset \mathfrak{h}^{\mathbb{T}} \text{ cpt Cartan}$$

$$\mathfrak{h}^{\mathbb{C}} \supset \max_{\text{abel split}} \mathfrak{h}^{\mathbb{C}} = \text{cen } \mathfrak{h}^{\mathbb{C}} \xrightarrow{\text{exp}} \mathfrak{h}^{\mathbb{C}} = \text{exp } \mathfrak{h}^{\mathbb{C}} = \text{cen } \mathfrak{h}^{\leq \mathbb{C}} \supset \text{abel split prim } \mathfrak{h}^{\mathbb{C}}$$

$$\mathfrak{h}^{\mathbb{C}} \text{ co-root} \left\{ \begin{array}{l} \mathfrak{h}^{\mathbb{C}} \\ \frac{1}{\mathfrak{h}^{\mathbb{C}}} \ni 1 \\ -\frac{1}{\mathfrak{h}^{\mathbb{C}}} \ni 1 \end{array} \right\} = \left\{ \begin{array}{l} \langle \mathfrak{h}^{\mathbb{C}} \rangle = \frac{-1}{\mathfrak{h}^{\mathbb{C}}} \\ \mathfrak{h}^{\mathbb{C}} \ni 1 \\ \langle \mathfrak{h}^{\mathbb{C}} \rangle = \frac{1}{\mathfrak{h}^{\mathbb{C}}} \\ \mathfrak{h}^{\mathbb{C}} \ni 1 \end{array} \right.$$

$$\mathfrak{h}^{\mathbb{C}} \supset \mathfrak{h}^{\mathbb{C}} \supset \mathfrak{h}^{\mathbb{C}} = \frac{\mathfrak{b} \in \mathfrak{h}^{\mathbb{C}}}{\bigwedge_{\mathfrak{b} \in \mathfrak{h}^{\mathbb{C}}} \mathfrak{b} \times \mathfrak{b} = \mathfrak{b} \mathfrak{b}}$$

$$\mathfrak{h}^{\mathbb{C}} \text{ co-root} \left\{ \begin{array}{l} \mathfrak{h}_{\mathbb{T}}^{\mathbb{C}} \\ \frac{1}{\mathfrak{h}_{\mathbb{T}}^{\mathbb{C}}} \\ -\frac{1}{\mathfrak{h}_{\mathbb{T}}^{\mathbb{C}}} \ni 1 \end{array} \right\} = \left\{ \begin{array}{l} \langle \mathfrak{h}_{\mathbb{T}}^{\mathbb{C}} \rangle = \frac{-1}{\mathfrak{h}_{\mathbb{T}}^{\mathbb{C}}} \\ \mathfrak{h}_{\mathbb{T}}^{\mathbb{C}} \ni 1 \\ \langle \mathfrak{h}_{\mathbb{T}}^{\mathbb{C}} \rangle = \frac{1}{\mathfrak{h}_{\mathbb{T}}^{\mathbb{C}}} \\ \mathfrak{h}_{\mathbb{T}}^{\mathbb{C}} \ni 1 \end{array} \right.$$

$$\mathfrak{h}^{\mathbb{C}} \supset \mathfrak{h}_{\mathbb{T}}^{\mathbb{C}} = \mathfrak{h}_{\mathbb{T}}^{\mathbb{C}} = \mathfrak{h}_{\mathbb{T}}^{\mathbb{C}}$$

$$\mathfrak{h}^{\mathbb{R}} = \mathfrak{h}_{\mathbb{T}}^{\mathbb{R}} = \frac{\mathfrak{h}_{\mathbb{T}}^{\mathbb{R}}}{\mathfrak{h}_{\mathbb{T}}^{\mathbb{R}}} = \frac{\mathfrak{h}^{\mathbb{R}}}{\mathfrak{h}_{\mathbb{T}}^{\mathbb{R}}} \times \mathfrak{h}_{\mathbb{T}}^{\mathbb{R}} \supset \max_{\text{abel}} \mathfrak{h}_{\mathbb{T}}^{\mathbb{R}} \text{ cpt}$$

$$\mathbb{H}^{\mathbb{C}} \setminus \mathbb{H}^{\mathbb{C}} = \underbrace{\mathbb{H}^{\mathbb{R}} \setminus \mathbb{H}^{\mathbb{T}}}_{\text{cpt}} \cup \underbrace{\mathbb{H}^{\mathbb{R}} \setminus \mathbb{H}^{\mathbb{R}}}_{\text{non-cpt}} \rightarrow 1$$

$$\mathbb{H}^{\mathbb{K}} \setminus \mathbb{H}^{\mathbb{K}} = \mathbb{H}^{\mathbb{C}}$$

$$\mathbb{H}^{\mathbb{C}} \cap \mathbb{H}^{\mathbb{R}} = \mathbb{H}^{\mathbb{R}}$$