$$\text{anti-cpt Cartan} \bigvee \text{abel split} \quad \begin{cases} \underline{\mathbb{h}}^{\mathbb{R}} \overset{\max}{\underset{\text{abel}}{\sqsupset}} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}} & = {}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{R}} \times {}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} \\ \underline{\mathbb{h}}^{\mathbb{R}i} \overset{\max}{\underset{\text{abel}}{\sqsupset}} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{T}} & = {}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{R}} \times {}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} i \\ \underline{\mathbb{h}}^{\mathbb{C}} \overset{\max}{\underset{\text{abel}}{\sqsupset}} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} & = {}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}} \times {}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}} \end{cases}$$

$$^{-}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} \supset {}^{\ddagger}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \frac{\alpha \in {}^{-}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{{}_{\circ}\underline{\mathbb{h}}_{\models}^{\mathbb{R}} | \alpha > 0}$$

$$\underline{\mathbb{h}}^{\mathbb{C}} = \begin{cases} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} \\ ^{\gtrless}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} & = \dfrac{{}^{\pm\alpha}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{\alpha \in {}^{\ddagger}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}} \end{cases} = \begin{cases} \underline{\mathbb{h}}_{\circ}^{\mathbb{C}} & = \begin{cases} {}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}} \\ ^{\gtrless}{}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}} \end{cases} = \dfrac{{}^{\pm\alpha}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{\alpha \in {}^{\ddagger}{}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}}} \\ ^{\gtrless}{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}} & = \dfrac{{}^{\pm\alpha}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{\alpha \in {}^{\ddagger}{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}}} \end{cases}$$

$$^{\alpha}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \underline{\mathbb{h}}^{\mathbb{C}} \blacktriangleleft^{\alpha}{}_{{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}} = \frac{\beta \in \underline{\mathbb{h}}^{\mathbb{C}}}{\bigwedge_{\gamma \in {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}} \beta \times \gamma = \widehat{\gamma \alpha \beta}}$$

$$\underline{\mathbb{h}}_{\circ}^{\mathbb{C}} \underset{\text{anti-root}}{\underline{\underline{\mathbb{C}}}} \begin{cases} {}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}} \\ \dfrac{{}^{\alpha}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{\alpha \in {}^{-}{}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}}} \end{cases} = \begin{cases} ^{<}{}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}} & = \dfrac{{}^{-\alpha}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{\alpha \in {}^{\ddagger}{}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}}} \\ ^{>}{}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}} & = \dfrac{{}^{\alpha}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{\alpha \in {}^{\ddagger}{}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}}} \end{cases}$$

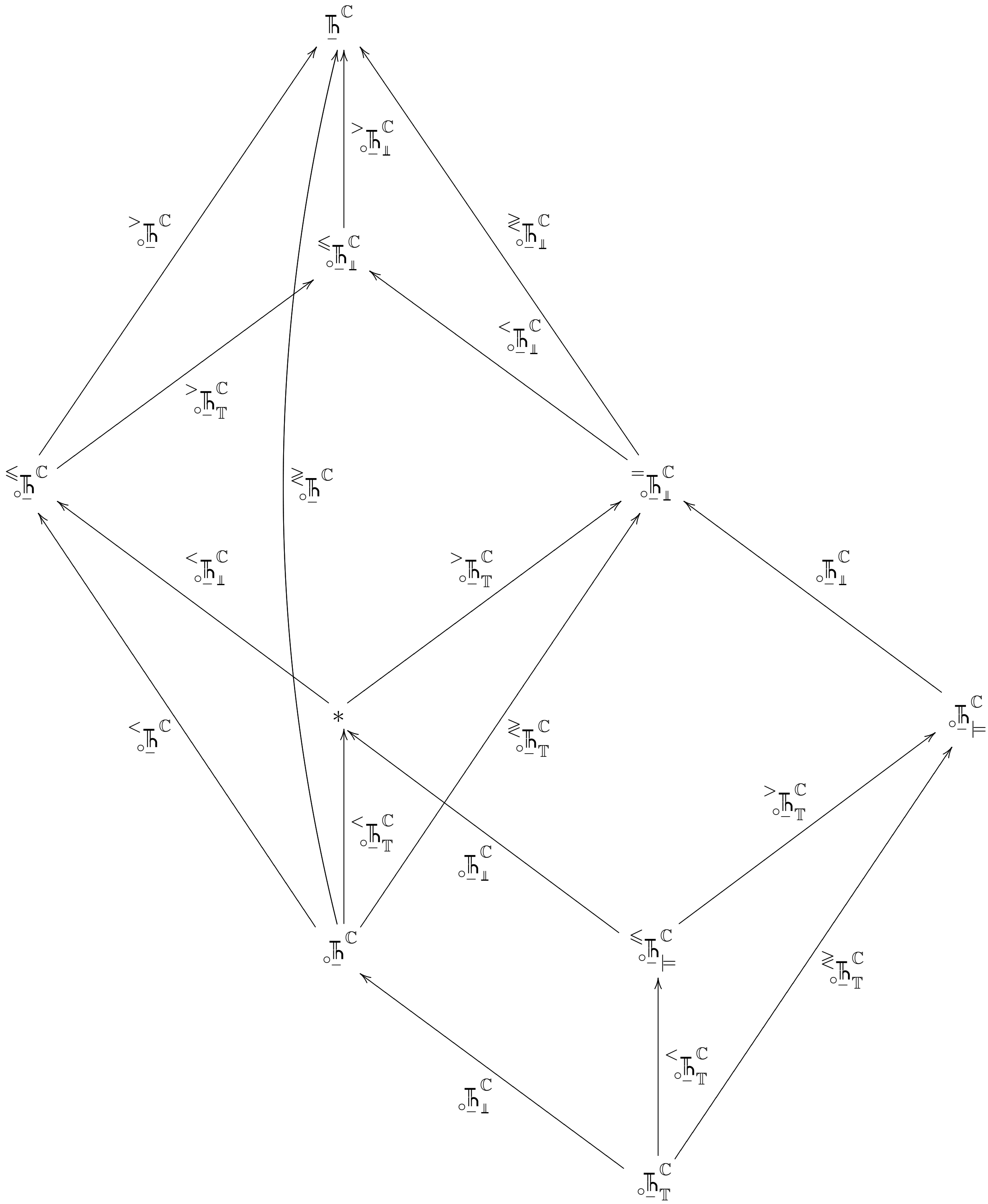
$$\text{prim } {}_{\circ}\underline{\mathbb{h}}_{\models}^{\mathbb{R}} \overset{\max}{\subset} \frac{\gamma \in {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}}{\bigwedge_{\alpha \neq 0} \gamma\alpha \neq 0} \text{ Weyl chamber}$$

$$^{\ddagger}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \begin{cases} ^{\ddagger}{}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}} & = \dfrac{\alpha \in {}^{\ddagger}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}} | \alpha = 0} \\ ^{\ddagger}{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}} & = \dfrac{\alpha \in {}^{\ddagger}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}} | \alpha \neq 0} \end{cases} \subset {}^{-}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \begin{cases} ^{-}{}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{C}} & = \dfrac{\alpha \in {}^{-}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}} | \alpha = 0} \\ ^{-}{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}} & = \dfrac{\alpha \in {}^{-}{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}} | \alpha \neq 0} \end{cases}$$

$$^{=}{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}} = \underline{\mathbb{h}}^{\mathbb{C}} \blacktriangleleft_{{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{C}}}$$

$$ {}_{\circ}\underline{\hbar}^{\mathbb{C}}_{\models} = \underline{\hbar}^{\mathbb{C}} \blacktriangleleft_{1} {}_{\circ}\underline{\hbar}^{\mathbb{C}}_{\perp\!\!\!\perp} $$

$$ {}^{\leqslant}{}_{\circ}\underline{\hbar}^{\mathbb{C}}_{\models} = \underline{\hbar}^{\mathbb{C}} \blacktriangleleft^{\subseteq}_{1} {}_{\circ}\underline{\hbar}^{\mathbb{C}}_{\perp\!\!\!\perp} $$



$$\underline{\mathbb{h}}^{\mathbb{C}} = \begin{cases} {}^{=}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \begin{cases} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} \\ \dfrac{{}^{1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{1 \in {}^{-}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}}} \end{cases} = \begin{cases} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\perp} \\ {}^{=}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} \end{cases} \\ {}^{\neq}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \dfrac{{}^{1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{1 \in {}^{-}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\perp}} \end{cases}$$

$${}^{=}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} = {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} \times \frac{{}^{1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{1 \in {}^{-}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}}} \sqsubset \underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}}$$

$${}^{=}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \underline{\mathbb{h}}^{\mathbb{C}} \blacktriangleright {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\perp} \sqsupset {}^{=}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} = \underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} \blacktriangleright {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\perp}$$

$$\begin{cases} \flat \in {}^{1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} \\ \flat \in {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\perp} \sqsubset {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} \end{cases} \Longrightarrow \flat \times \flat = \underbrace{\flat 1 \flat} \Longrightarrow {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\perp} | 1 = 0 \Leftrightarrow {}^{1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} \sqsubset \underline{\mathbb{h}}^{\mathbb{C}} \blacktriangleright {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\perp} \overset{{}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\perp} \sqsubset {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{\Longleftrightarrow} {}^{1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} \sqsubset \underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} \blacktriangleright {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\perp}$$

$$\underline{\mathbb{h}}^{\mathbb{C}} = \begin{cases} {}^{\leqslant}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \begin{cases} {}^{<}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \dfrac{{}^{-1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{1 \in {}^{\ddagger}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}} \\ {}^{=}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} \end{cases} \\ {}^{>}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} = \dfrac{{}^{1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{1 \in {}_{1}\underline{\mathbb{h}}^{\mathbb{C}\ddagger}\bigcirc} \end{cases}$$

$$\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} = \begin{cases} {}^{\leqslant}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} \underset{\text{minibolic}}{\underline{\mathbb{C}}} \begin{cases} {}^{<}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} = \dfrac{{}^{-1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{1 \in {}^{\ddagger}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}}} \\ {}^{=}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} \end{cases} \\ {}^{>}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} = \dfrac{{}^{1}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{1 \in {}^{\ddagger}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}}} \end{cases}$$

$${}^{\leqslant}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} \neg {}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}} = {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}_{\mathbb{T}} \neg {}_{\circ}\underline{\mathbb{h}}_{\text{cpt}}{}^{\mathbb{R}}_{\mathbb{T}} \to {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}} \neg \underline{\mathbb{h}}^{\mathbb{R}} \to \left({}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}_{\mathbb{T}} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}_{\perp} \right) \neg \underline{\mathbb{h}}^{\mathbb{R}}$$

$$\underline{{}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}_{\mathbb{T}} \neg {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}_{\mathbb{T}}} = {}^{>}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}_{\mathbb{T}}$$