

2^ℓ choices $\square \subset \square \subset \square \xrightarrow{\text{simple}} \square \xrightarrow{\dagger} \square^{\mathbb{C}}$ pos : $|\blacksquare| = \ell$

$$0 = \blacksquare \underline{K}^{\mathbb{C}} \subset \square \underline{K}^{\mathbb{C}} = \frac{\blacksquare \in \square \underline{K}^{\mathbb{C}}}{\blacksquare | \square = 0} \subset \square \underline{K}^{\mathbb{C}} \text{ subtorus}$$

$$\square \underline{K}^{\mathbb{I}\mathbb{C}} = \square \underline{K}^{\mathbb{C}} \times \square \underline{K}^{\mathbb{X}\mathbb{C}}$$

$$\square \underline{K}^{\mathbb{X}\mathbb{C}} = \frac{\square \underline{K}^{\mathbb{I}\mathbb{C}} \subset \square \underline{K}^{\mathbb{C}}}{\square \underline{K}^{\mathbb{I}\mathbb{C}} \ni \mathbf{1} \in \langle \square \rangle} = \square \underline{K}^{\mathbb{X}\mathbb{C}} \times \square \underline{K}^{\mathbb{X}\mathbb{C}}$$

$$\square \underline{K}^{\mathbb{I}\mathbb{C}} = \square \underline{K}^{\mathbb{X}\mathbb{C}}$$

$$\square \underline{K}^{\mathbb{X}\mathbb{C}} = \frac{\square \underline{K}^{\mathbb{I}\mathbb{C}} \subset \square \underline{K}^{\mathbb{C}}}{\square \underline{K}^{\mathbb{I}\mathbb{C}} \ni \mathbf{1} \in \langle \square \rangle} = \square \underline{K}^{\mathbb{X}\mathbb{C}} \times \square \underline{K}^{\mathbb{X}\mathbb{C}}$$

$$\mathbb{R} \square \underline{K}^{\mathbb{I}\mathbb{C}} = \square \underline{K}^{\mathbb{I}\mathbb{C}} \times \square \underline{K}^{\mathbb{I}\mathbb{C}} = \square \underline{K}^{\mathbb{I}\mathbb{C}} \times \square \underline{K}^{\mathbb{X}\mathbb{C}}$$

$$= \square \underline{K}^{\mathbb{C}} \times \overbrace{\square \underline{K}^{\mathbb{X}\mathbb{C}} \times \square \underline{K}^{\mathbb{X}\mathbb{C}}}^{\mathbb{R} \square \underline{K}^{\mathbb{X}\mathbb{C}}}$$

$$\mathbb{R} \square \underline{K}^{\mathbb{X}\mathbb{C}} = \frac{\mathbb{R} \square \underline{K}^{\mathbb{I}\mathbb{C}}}{\square \underline{K}^{\mathbb{I}\mathbb{C}} \ni \mathbf{1} \in \langle \square \rangle} = \mathbb{R} \square \underline{K}^{\mathbb{X}\mathbb{C}} \times \mathbb{R} \square \underline{K}^{\mathbb{X}\mathbb{C}}$$

$$\begin{aligned}
\underline{K}^C &= \underline{K}^I \times \underline{K}^M = \overbrace{\underline{K}^I \times \underline{K}^M}^{= \underline{K}^X} \times \underline{K}^X \\
&= \underline{K}^I \times \overbrace{\underline{K}^X \times \underline{K}^M}^{= \underline{K}^X}
\end{aligned}$$

$$\underline{K}^M = \frac{\underline{K}^I \subset \underline{K}^C}{\langle \square \rangle \not\cong 1 \in \underline{K}^I} = \underline{K}^X \times \underline{K}^X$$

$$\begin{aligned}
\underline{K}^C &= \underline{K}^I \times \underline{K}^M \\
&= \underline{K}^I \times \underline{K}^X \times \underline{K}^X
\end{aligned}$$

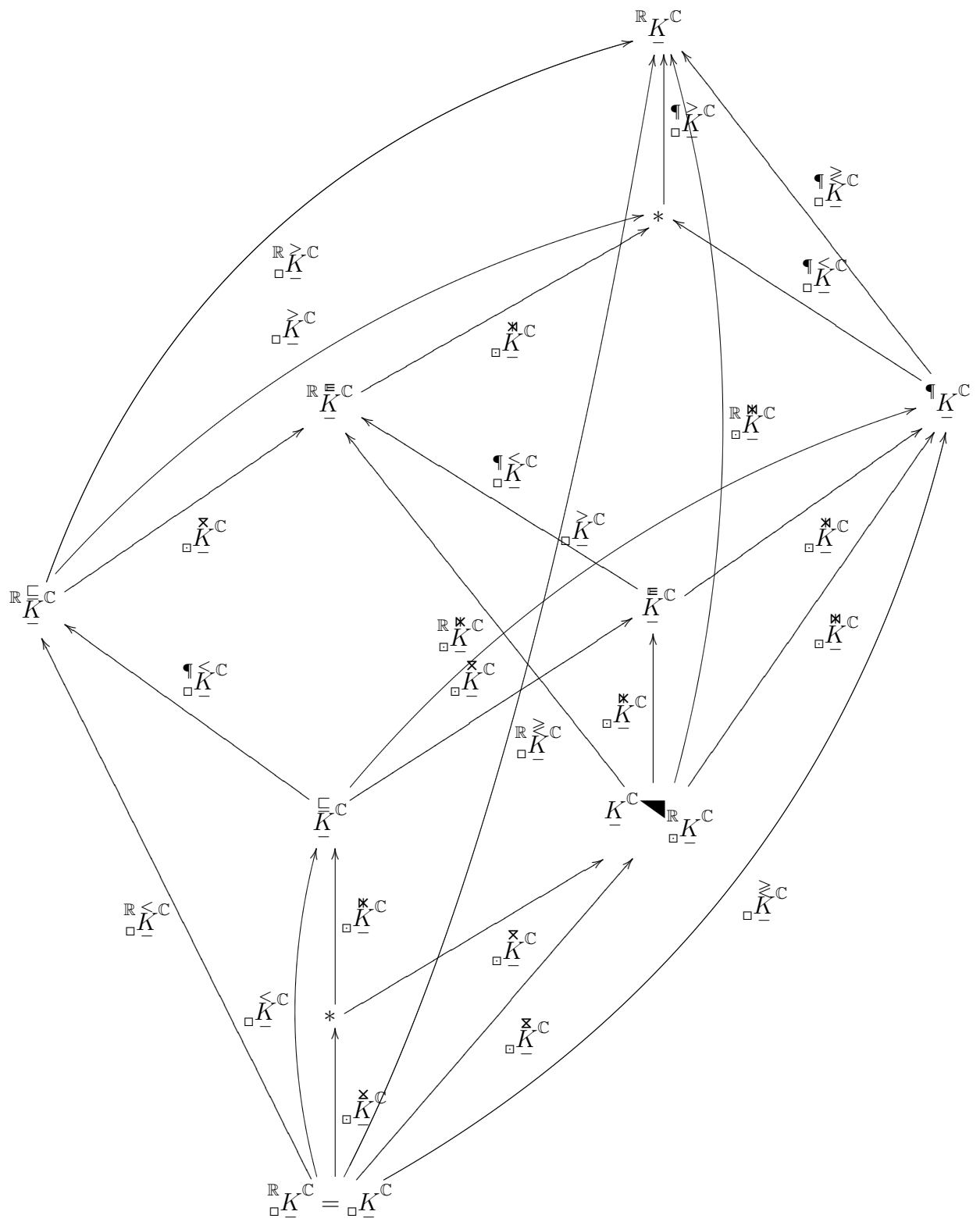
$$\underline{K}^M = \frac{\underline{K}^I \subset \underline{K}^C}{\langle \square \rangle \not\cong 1 \in \underline{K}^I} = \underline{K}^X \times \underline{K}^X$$

$$\begin{aligned}
\underline{\mathbb{R}} \underline{K}^C &= \underline{\mathbb{R}} \underline{K}^I \times \overbrace{\underline{\mathbb{R}} \underline{K}^M \times \underline{\mathbb{1}} \underline{\mathbb{R}} \underline{K}^M}^{\underline{\mathbb{R}} \underline{K}^C} = \overbrace{\underline{\mathbb{R}} \underline{K}^I \times \underline{\mathbb{R}} \underline{K}^X}^{\underline{\mathbb{R}} \underline{K}^C} \times \underline{\mathbb{R}} \underline{K}^X \\
&= \overbrace{\underline{\mathbb{R}} \underline{K}^I \times \underline{\mathbb{R}} \underline{K}^M}^{\underline{K}^C} \times \overbrace{\underline{\mathbb{1}} \underline{\mathbb{R}} \underline{K}^X \times \underline{\mathbb{1}} \underline{\mathbb{R}} \underline{K}^M}^{\underline{\mathbb{1}} \underline{K}^C} \\
&= \underline{\mathbb{R}} \underline{K}^C \times \overbrace{\overbrace{\underline{\mathbb{R}} \underline{K}^X \times \underline{\mathbb{R}} \underline{K}^M}^{\underline{\mathbb{R}} \underline{K}^C} \times \overbrace{\underline{\mathbb{1}} \underline{\mathbb{R}} \underline{K}^X \times \underline{\mathbb{1}} \underline{\mathbb{R}} \underline{K}^M}^{\underline{\mathbb{1}} \underline{K}^C}}^{\underline{\mathbb{R}} \underline{K}^C} = \overbrace{\underline{\mathbb{R}} \underline{K}^X \times \underline{\mathbb{1}} \underline{\mathbb{R}} \underline{K}^X}^{\underline{\mathbb{R}} \underline{K}^C} \times \overbrace{\underline{\mathbb{R}} \underline{K}^M \times \underline{\mathbb{1}} \underline{\mathbb{R}} \underline{K}^M}^{\underline{\mathbb{R}} \underline{K}^C}
\end{aligned}$$

$$\underline{\mathbb{R}} \underline{K}^M = \frac{\underline{\mathbb{R}} \underline{K}^1}{\langle \square \rangle \not\cong \underline{\mathbb{1}} \in \underline{\mathbb{R}} \underline{K}^{\dagger}} = \underline{\mathbb{R}} \underline{K}^X \times \underline{\mathbb{R}} \underline{K}^X$$

$$\underline{\mathbb{R}} \underline{K}^X \times \underline{\mathbb{R}} \underline{K}^X = \underline{\mathbb{R}} \underline{K}^X: \quad \underline{\mathbb{R}} \underline{K}^X \times \underline{\mathbb{R}} \underline{K}^C = \underline{\mathbb{R}} \underline{K}^C$$

$$\underline{\mathbb{R}} \underline{K}^C \supset \begin{cases} \underline{\mathbb{R}} \underline{K}^<C \\ \underline{\mathbb{R}} \underline{K}^=C \\ \underline{\mathbb{R}} \underline{K}^>C \end{cases} = \begin{cases} \underline{\mathbb{R}} \underline{K}^{\leq C} \\ \underline{\mathbb{R}} \underline{K}^{\geq C} \end{cases}$$



$$\mathbb{R}_{\square}^{\geq C} \underline{K} = \frac{\mathbb{R} \underline{K}^C}{\langle \square \rangle \not\cong \mathbf{1} \in \mathbb{H}^C} \text{ nilp rad}$$

$$\mathbb{R}_{\square}^{\leq C} \underline{K} = \mathbb{R}_{\square}^{\leq C} \underline{K} \times \mathbb{R}_{\square}^{\leq C} \underline{K}$$

