$${}_{\square}\underline{K}^{\mathbb{C}} \underset{\mathrm{Car}}{\sqsubset} {}^{\mathbb{R}}\underline{K}^{\mathbb{C}}$$

$${}_{\square}\underline{K}^{\mathbb{C}} \underset{\mathrm{Car}}{\sqsubset} \underline{K}^{\mathbb{C}}$$

$${}^{\mathbb{R}}_{\square}\overset{\blacktriangleleft}{\underline{K}}^{\mathbb{C}} \sqsubset \begin{cases} \underline{K}^{\mathbb{C}} \\ {}^{\P}\underline{K}^{\mathbb{C}} \end{cases}$$

$${}^{\mathbb{R}}_{\square}\overset{\blacktriangleleft}{\underline{K}}^{\mathbb{C}} \ni \mathfrak{b} = \natural + \sharp \Longrightarrow \natural \in {}^{\mathbb{R}}_{\square}\overset{\blacktriangleleft}{\underline{K}}^{\mathbb{C}} \ni \sharp: \quad {}^{\mathbb{R}}_{\square}\underline{K}^{\mathbb{C}} \ni \flat$$

$$\natural \divideontimes \flat + \sharp \divideontimes \flat = \underbrace{\natural + \sharp} \divideontimes \flat = \underbrace{\flat\blacktriangleleft}\underbrace{\natural + \sharp} = \underbrace{\flat\blacktriangleleft}\natural + \underbrace{\flat\blacktriangleleft}\sharp \Longrightarrow \natural \divideontimes \flat = \underbrace{\flat\blacktriangleleft}\natural: \quad \sharp \divideontimes \flat = \underbrace{\flat\blacktriangleleft}\sharp$$

$${}_{\square}\overset{\intercal}{\underline{K}}^{\mathbb{C}} = \underline{K}^{\mathbb{C}} \blacktriangleright_{{}_{\square}\underline{K}^{\mathbb{C}}} = \frac{\mathfrak{b} \in \underline{K}^{\mathbb{C}}}{\mathfrak{b} \divideontimes {}_{\square}\underline{K}^{\mathbb{C}} = 0} = {}_{\square}\underline{K}^{\mathbb{C}}$$

$${}^{\mathbb{R}}_{\square}\overset{\intercal}{\underline{K}}^{\mathbb{C}} = {}^{\mathbb{R}}\underline{K}^{\mathbb{C}} \blacktriangleright_{{}^{\mathbb{R}}_{\square}\underline{K}^{\mathbb{C}}} = \frac{\mathfrak{b} \in {}^{\mathbb{R}}\underline{K}^{\mathbb{C}}}{\mathfrak{b} \divideontimes {}_{\square}\underline{K}^{\mathbb{C}} = 0} = {}_{\square}\underline{K}^{\mathbb{C}}$$

$$\underline{K}^{\mathbb{C}} = {}_{\square}\underline{K}^{\mathbb{C}} \times {}_{\square}\overset{\bowtie}{\underline{K}}^{\mathbb{C}} = \overbrace{{}_{\square}\underline{K}^{\mathbb{C}} \times {}_{\square}\overset{\ltimes}{\underline{K}}^{\mathbb{C}}}^{{}_{\square}\overset{\ltimes}{\underline{K}}^{\mathbb{C}}} \times {}_{\square}\overset{\rtimes}{\underline{K}}^{\mathbb{C}}$$

$${}_{\square}\overset{\bowtie}{\underline{K}}^{\mathbb{C}} = {}^{\mathbb{R}}_{\square}\overset{\bowtie}{\underline{K}}^{\mathbb{C}} = \frac{{}_{\square}\overset{\blacktriangleleft}{\underline{K}}^{\mathbb{C}} = {}^{\mathbb{R}}_{\square}\overset{\blacktriangleleft}{\underline{K}}^{\mathbb{C}} \sqsubset \underline{K}^{\mathbb{C}}}{\blacktriangleleft \in {}_{\square}\overset{\ddagger}{\underline{K}}^{\mathbb{C}}} = {}_{\square}\overset{\ltimes}{\underline{K}}^{\mathbb{C}} \times {}_{\square}\overset{\rtimes}{\underline{K}}^{\mathbb{C}}$$

$${}^{\P}\underline{K}^{\mathbb{C}} = {}^{\P}_{\square}\overset{\bowtie}{\underline{K}}^{\mathbb{C}}$$

$${}^{\P}_{\square}\overset{\bowtie}{\underline{K}}^{\mathbb{C}} = \frac{{}^{\mathbb{R}}_{\square}\overset{\blacktriangleleft}{\underline{K}}^{\mathbb{C}} \sqsubset {}^{\P}\underline{K}^{\mathbb{C}}}{\blacktriangleleft \in {}^{\mathbb{R}}_{\square}\overset{\ddagger}{\underline{K}}^{\mathbb{C}}} = {}^{\P}_{\square}\overset{\ltimes}{\underline{K}}^{\mathbb{C}} \times {}^{\P}_{\square}\overset{\rtimes}{\underline{K}}^{\mathbb{C}}$$

$${}^{\mathbb{R}}_{\square}\overset{\perp\!\!\!\perp}{\underline{K}}^{\mathbb{C}} = {}^{\mathbb{R}}\underline{K}^{\mathbb{C}} \blacktriangleright_{{}^{\P}_{\square}\underline{K}^{\mathbb{C}}} = {}^{\mathbb{R}}\underline{K}^{\mathbb{C}} = {}^{\mathbb{R}}_{\square}\overset{\perp}{\underline{K}}^{\mathbb{C}} \times \underbrace{{}^{\P}_{\square}\underline{K}^{\mathbb{C}}}_{=0} = {}^{\mathbb{R}}_{\square}\overset{\perp}{\underline{K}}^{\mathbb{C}}$$

$$
{}^{\mathbb{R}}\underline{K}^{\mathbb{C}} = {}_{\square}\underline{K}^{\mathbb{C}} \times \overbrace{{}_{\square}\underline{\overset{\bowtie}{K}}^{\mathbb{C}} \times {}_{\square}^{\P}\underline{\overset{\bowtie}{K}}^{\mathbb{C}}}^{{}^{\mathbb{R}}_{\square}\underline{\overset{\bowtie}{K}}^{\mathbb{C}}} = \overbrace{{}_{\square}\underline{K}^{\mathbb{C}} \times {}^{\mathbb{R}}_{\square}\underline{\overset{\ltimes}{K}}^{\mathbb{C}}}^{{}^{\mathbb{R}}_{\square}\underline{\overset{\ltimes}{K}}^{\mathbb{C}}} \times {}^{\mathbb{R}}_{\square}\underline{\overset{\rtimes}{K}}^{\mathbb{C}}
$$

$$
= \overbrace{{}_{\square}\underline{K}^{\mathbb{C}} \times {}_{\square}\underline{\overset{\bowtie}{K}}^{\mathbb{C}}}^{\underline{K}^{\mathbb{C}}} \times \overbrace{{}^{\P}_{\square}\underline{\overset{\bowtie}{K}}^{\mathbb{C}}}^{{}^{\P}\underline{K}^{\mathbb{C}}}
$$

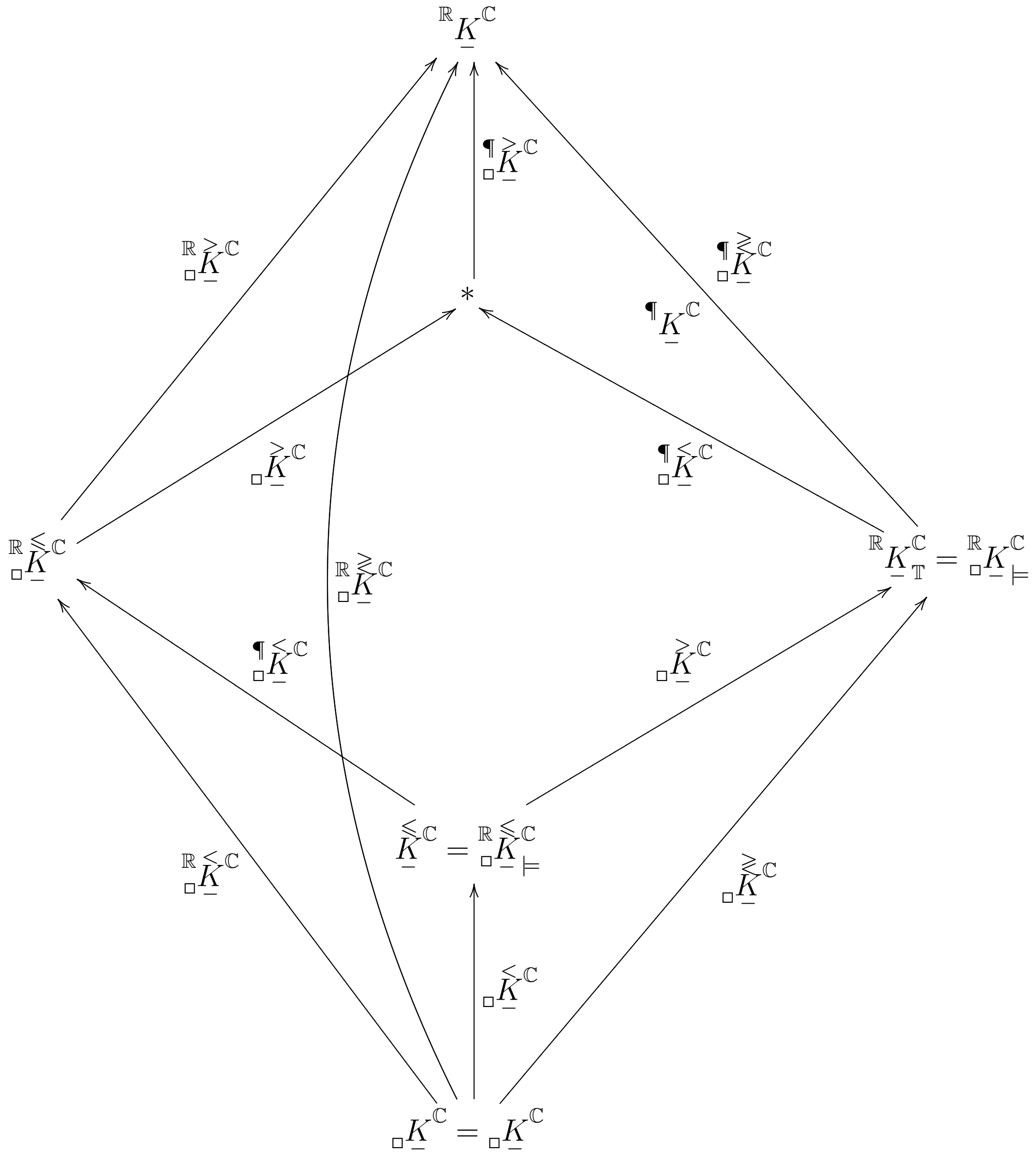
$$
{}^{\mathbb{R}}_{\square}\underline{\overset{\sharp}{K}}^{\mathbb{C}} \ni \text{ⅎ} \Longrightarrow {}^{\mathbb{R}}_{\square}\underline{\overset{\text{ⅎ}}{K}}^{\mathbb{C}} = \frac{\flat \in {}^{\mathbb{R}}\underline{K}^{\mathbb{C}}}{\bigwedge_{\flat \in {}^{\mathbb{R}}_{\square}\underline{K}^{\mathbb{C}}} \flat \ast \flat = \underbrace{\flat\,\text{ⅎ}}\,\flat}
$$

$$
{}^{\mathbb{R}}_{\square}\underline{\overset{\bowtie}{K}}^{\mathbb{C}} = \frac{{}^{\mathbb{R}}_{\square}\underline{\overset{\text{ⅎ}}{K}}^{\mathbb{C}}}{\text{ⅎ} \in {}_{\square}\underline{\overset{\ddagger}{K}}^{\mathbb{C}}} = \overbrace{{}_{\square}\underline{\overset{\ltimes}{K}}^{\mathbb{C}} \times {}^{\P}_{\square}\underline{\overset{\ltimes}{K}}^{\mathbb{C}}}^{{}^{\mathbb{R}}_{\square}\underline{\overset{\ltimes}{K}}^{\mathbb{C}}} \times \overbrace{{}_{\square}\underline{\overset{\rtimes}{K}}^{\mathbb{C}} \times {}^{\P}_{\square}\underline{\overset{\rtimes}{K}}^{\mathbb{C}}}^{{}^{\mathbb{R}}_{\square}\underline{\overset{\rtimes}{K}}^{\mathbb{C}}}
$$

$$
{}^{\mathbb{R}}_{\square}\overset{\ltimes}{K}^{\mathbb{C}} \urcorner {}^{\mathbb{R}}K^{\mathbb{C}} = \underbrace{{}_{\square}K \urcorner {}^{\mathbb{T}}K}_{\text{cpt}} \supset \underbrace{{}_{\square}K \urcorner {}^{\mathbb{R}}K}_{\text{non-cpt}} \to 1
$$

$$
\underline{{}_{\square}K \urcorner {}^{\mathbb{K}}K} = {}^{\mathbb{R}}_{\square}\underline{\overset{\ast}{K}}^{\mathbb{C}}
$$

$$
{}^{\mathbb{R}}_{\square}\underline{\overset{\ltimes}{K}}^{\mathbb{C}} \cap {}^{\mathbb{R}}\underline{K} = {}_{\square}\underline{K}
$$



$$ {}^{\mathbb{R}}\underline{K}^{\mathbb{C}} \;\underset{\text{abel split}}{\overset{\max}{\sqsupset}}\; {}_{\square}\underline{K}^{\mathbb{C}} = \operatorname{cen} {}^{\mathbb{R}}_{\square}\underline{K}^{\lessdot\mathbb{C}} \xrightarrow{\exp} {}_{\square}\mathbb{h}^{\mathbb{C}} = \exp {}_{\square}\underline{K}^{\mathbb{C}} = \operatorname{cen} {}_{\square}\mathbb{h}^{\lesseqgtr\mathbb{C}} \;\underset{\text{abel split prim}}{\sqsubset}\; \mathbb{h}^{\mathbb{C}} $$

$$ \mathbb{h}^{\mathbb{C}} \sqsupset \begin{cases} {}_{\square}\mathbb{h}^{\lessdot\mathbb{C}} \\ \\ {}_{\square}\mathbb{h}^{>\mathbb{C}} \end{cases} \;\underset{\text{minibolic}}{\underline{\underline{\mathbb{C}}}}\; \begin{cases} {}_{\square}\mathbb{h}^{<\mathbb{C}} \\ {}_{\square}\mathbb{h}^{=\mathbb{C}} \end{cases} $$

$$ \underbrace{\mathbb{h}_{[-}^{\mathbb{R}\mathbb{R}}{}_{\square}\underline{K}^{\mathbb{C}}]8>}_{\mathbb{N}\text{ abel}} = {}_{8}\mathbb{h}^{\mathbb{R}}_{-} \lessdot \times {}_{8}\mathbb{h}^{\mathbb{R}}_{-} \lessdot \xrightarrow{\exp} \underbrace{{}_{\square}\mathbb{h}^{>\mathbb{C}}}_{\mathbb{N}\text{ abel prim}} = {}_{\square}\mathbb{h}^{\lessdot\mathbb{C}} \times {}_{\square}\mathbb{h}^{\lessdot\mathbb{C}} $$

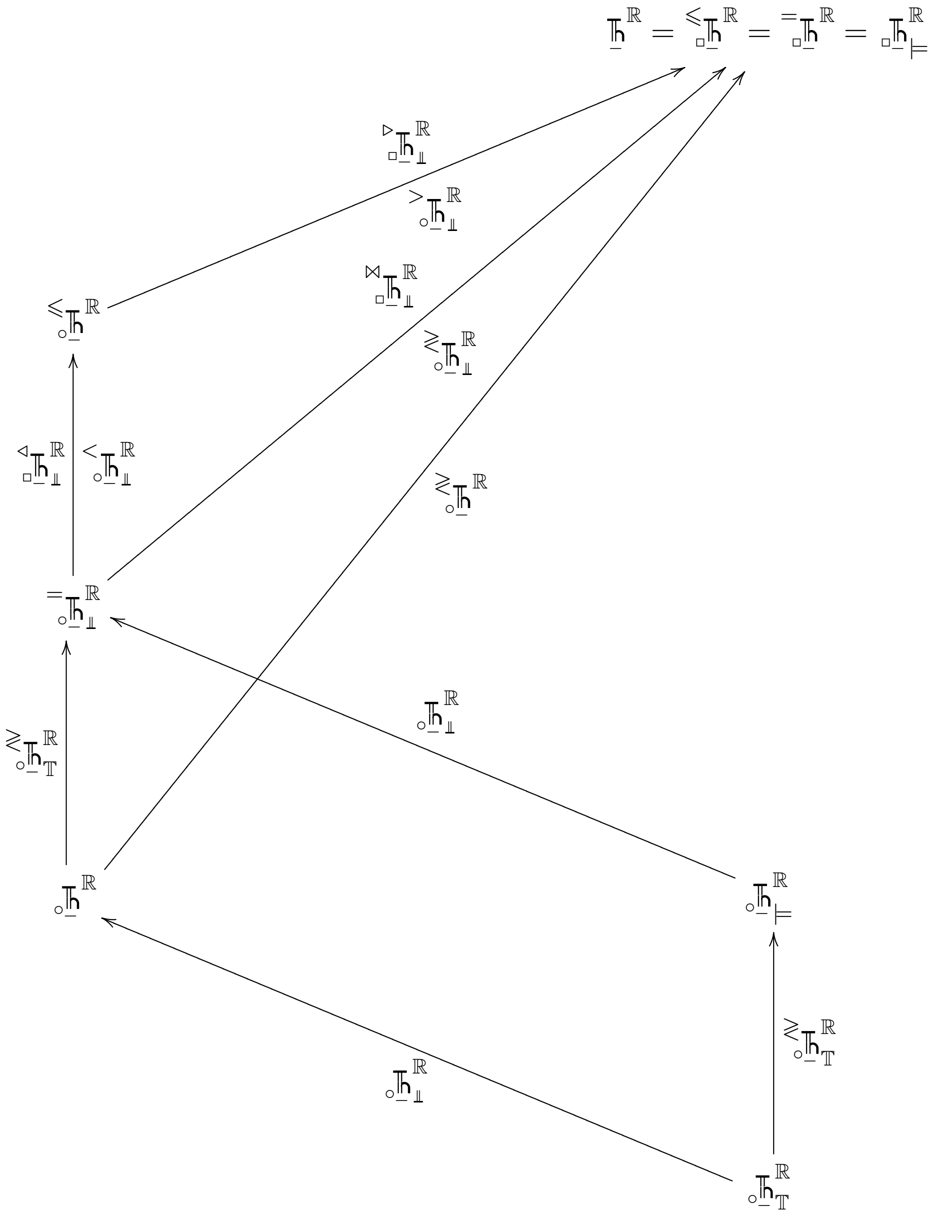
$$ {}^{\mathbb{R}}\underline{K}^{\mathbb{C}} \overset{\max}{\sqsupset} \underbrace{{}_{8}\underline{\mathbb{h}}^{\mathbb{R}} \leqslant}_{2^{\mathbb{N}}\ \text{abel}} = {}_{8}\underline{\mathbb{h}}^{\mathbb{R}} < \ltimes \xrightarrow{\exp} \underline{\mathbb{h}}^{\mathbb{R}} \underbrace{{}_{\square}\overset{\geqslant}{\mathbb{h}}^{\mathbb{C}}}_{2^{\mathbb{N}}\ \text{abel prim}} = {}_{\square}\overset{<}{\mathbb{h}}^{\mathbb{C}} \ltimes {}_{\square}\mathbb{h}^{\mathbb{C}} \overset{\max}{\sqsubset} \mathbb{h}^{\mathbb{C}} $$

old

$$ \text{prim} \ \square \subset {}^{\ddagger}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}_{\perp\!\!\perp} \ \text{pos nc roots} $$

$$ |\,\square\,| = r $$

$$ {}_{\square}\underline{\mathbb{h}}^{\mathbb{R}}_{\perp\!\!\perp} = {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}_{\perp\!\!\perp} \blacktriangleleft_{\square} = \frac{\flat \in {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}_{\perp\!\!\perp}}{\flat |\,\square = 0} = 0 $$



$$\underline{\mathbb{h}}^{\mathbb{R}} \;\underset{\text{co-root}}{\overline{\overline{\mathbb{R}}}}\; \underline{\mathbb{h}}^{\mathbb{R}} \blacktriangleleft {}_{\square}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} = \begin{cases} \underline{\mathbb{h}}^{\mathbb{R}} \blacktriangleleft {}_{\square}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} & = \begin{cases} \underline{\mathbb{h}}^{\mathbb{R}} \blacktriangleleft {}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} & = \begin{cases} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}} \\ \dfrac{}{{}^{\ddagger}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{R}} \ni \text{Ɦ}} {}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} \end{cases} \\ \dfrac{{}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}}}{{}^{-}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} \ni \text{Ɦ} \in <\square>} \end{cases} \\ \dfrac{{}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}}}{{}^{-}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} \ni \text{Ɦ} \notin <\square>} & = 0 \end{cases} = \begin{cases} \underline{\mathbb{h}}^{\mathbb{R}} \blacktriangleleft {}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} & = \begin{cases} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}} \\ \dfrac{{}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}}}{{}^{\ddagger}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{R}} \ni \text{Ɦ}} \end{cases} \\ \dfrac{{}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}}}{{}^{-}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} \ni \text{Ɦ}} \end{cases}$$

$$\underline{\mathbb{h}}^{\mathbb{R}} = {}^{=}_{\square}\underline{\mathbb{h}}^{\mathbb{R}} = \begin{cases} {}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}} \\ {}^{0}_{\square}\underline{\mathbb{h}}^{\mathbb{R}} \end{cases} = \dfrac{{}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}}{{}^{-}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} \ni \text{Ɦ}} = \underline{\mathbb{h}}^{\mathbb{R}} \cap \dfrac{{}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}}}{{}^{-}_{\circ}\underline{\mathbb{h}}^{\mathbb{C}} \ni \text{Ɦ}} = \begin{cases} {}^{0}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}} \\ \dfrac{{}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}}{{}^{-}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} \ni \text{Ɦ}} \end{cases} = \begin{cases} {}^{=}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}} \\ \dfrac{{}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}}{{}^{-}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} \ni \text{Ɦ}} \end{cases} = \begin{cases} {}^{=}_{\circ}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{R}} \\ {}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\mathbb{R}} \\ \dfrac{{}^{\text{Ɦ}}_{\circ}\underline{\mathbb{h}}^{\mathbb{R}}}{{}_{\circ}\underline{\mathbb{h}}_{\perp\!\!\!\perp}^{\overline{-}\,\mathbb{R}} \ni \text{Ɦ}} \end{cases}$$

$$\text{cpt}\;\; {}_{\square}\underline{\mathbb{h}}^{\mathbb{R}} = {}_{\square}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{R}} \underset{\text{abel}}{\overset{\text{max}}{\sqsubset}} \underline{\mathbb{h}}^{\mathbb{R}} = {}^{=}_{\square}\underline{\mathbb{h}}^{\mathbb{R}} = {}^{=}_{\square}\underline{\mathbb{h}}_{\mathbb{T}}^{\mathbb{R}}$$

$${}^{\gtrless}_{\square}\underline{\mathbb{h}}^{\mathbb{R}} = 0$$

$${}_{\square}\underline{\overset{\lessgtr}{\mathbb{h}}}^{\mathbb{R}} = {}_{\square}\underline{\overset{\equiv}{\mathbb{h}}}^{\mathbb{R}} = \underline{\mathbb{h}}^{\mathbb{R}}\, \text{full}\underset{\mathbb{R}}{\text{b}}\text{olic}$$

$${}_{\bigcirc}\underline{\overset{\geq}{\mathbb{h}}}^{\mathbb{R}} = 0$$