

$$\lambda \in \mathfrak{h}^*_{\mathbb{C}}$$

$$\text{pure } \mathfrak{h}^*_{K^\lambda} \in K \cap \mathfrak{h}^*_{K^\lambda} \cap K \cap \mathfrak{h}^*_{\infty}$$

$$\mathfrak{h}^*_{gK^\lambda} = \mathfrak{h}^*_{KgK^\lambda} = \int_{dk}^K (kg)^{\rho + \lambda} = \int_{dk}^K \overbrace{N_{\mathfrak{h}^*_{K^\lambda}}(kg)}^{\rho + \lambda}$$

$$\lambda \in W \cap \mathfrak{h}^*_{\mathbb{C}}$$

$$\mathfrak{h}^*_{K^\lambda} = \mathfrak{h}^*_{K^\lambda} \Leftrightarrow \lambda \in W \cap \mathfrak{h}^*_{\mathbb{C}}$$