$$\left(\frac{d}{dt}\right)^{\ell} e^{\lambda t}\, {}^{t}u = e^{\lambda t} \sum_{0\leqslant k\leqslant \ell} \begin{bmatrix}\ell\\ k\end{bmatrix} \lambda^{\ell-k}\, {}^{t}_{k}\underline{u}$$

$$\left(\frac{d}{dt}\right)^{\ell+1} e^{\lambda t}\, {}^{t}u = \frac{d}{dt}\left(\frac{d}{dt}\right)^{\ell} e^{\lambda t}\, {}^{t}u \underset{\text{ind}}{=} \frac{d}{dt}\, e^{\lambda t} \sum_{0\leqslant k\leqslant \ell} \begin{bmatrix}\ell\\ k\end{bmatrix} \lambda^{\ell-k}\, {}^{t}_{k}\underline{u}$$

$$= \lambda e^{\lambda t} \sum_{0\leqslant k\leqslant \ell} \begin{bmatrix}\ell\\ k\end{bmatrix} \lambda^{\ell-k}\, {}^{t}_{k}\underline{u} + e^{\lambda t} \sum_{0\leqslant k\leqslant \ell} \begin{bmatrix}\ell\\ k\end{bmatrix} \lambda^{\ell-k}\, {}^{t}_{k+1}\underline{u} = e^{\lambda t}\, \overbrace{\lambda \sum_{0\leqslant m\leqslant \ell} \begin{bmatrix}\ell\\ m\end{bmatrix} \lambda^{\ell-m}\, {}^{t}_{m}\underline{u} + \sum_{0\leqslant k\leqslant \ell} \begin{bmatrix}\ell\\ k\end{bmatrix} \lambda^{\ell-k}\, {}^{t}_{k+1}\underline{u}}$$

$$= e^{\lambda t}\, \overbrace{\sum_{0\leqslant m\leqslant \ell} \begin{bmatrix}\ell\\ m\end{bmatrix} \lambda^{\ell+1-m}\, {}^{t}_{m}\underline{u} + \sum_{1\leqslant m\leqslant \ell+1} \begin{bmatrix}\ell\\ m-1\end{bmatrix} \lambda^{\ell+1-m}\, {}^{t}_{m}\underline{u}}$$

$$= e^{\lambda t}\, \overbrace{\sum_{1\leqslant m\leqslant \ell} \underbrace{\begin{bmatrix}\ell\\ m\end{bmatrix} + \begin{bmatrix}\ell\\ m-1\end{bmatrix}}_{=\begin{bmatrix}\ell+1\\ m\end{bmatrix}} \lambda^{\ell+1-m}\, {}^{t}_{m}\underline{u} + \lambda^{\ell+1}\, {}^{t}u + {}^{t}_{\ell+1}\underline{u}} = e^{\lambda t} \sum_{0\leqslant k\leqslant \ell+1} \begin{bmatrix}\ell+1\\ k\end{bmatrix} \lambda^{\ell+1-k}\, {}^{t}_{k}\underline{u}$$