

$$\begin{aligned}
{}_z\mathbb{R}K^\lambda &= \int_{dv}^S \left( \frac{1 - z\bar{z}^*}{|z - v|^2} \right)^{1-i\lambda} \\
&= {}_{z\bar{z}^*} \left[ \begin{matrix} 1+i\lambda & 1+i\lambda \\ & 1 \end{matrix} \right] \overbrace{\left( \frac{1 - z\bar{z}^*}{|z - v|^2} \right)^{1-i\lambda}} = \frac{-z\bar{z}^*}{1 - z\bar{z}^*} \left[ \begin{matrix} 1+i\lambda & 1-i\lambda \\ & 1 \end{matrix} \right]
\end{aligned}$$