$$\mathbb{1} \ltimes \mathbb{1} \rtimes \mathbb{1} \xleftarrow[\text{bimod}]{} \mathbb{1}$$

$$\mathbb{1}_{m} \triangleleft_{\mathbb{1}} = \left\{ \mathbb{1} \xleftarrow[\text{lin}]{\mathbb{L}} \overset{m}{\otimes} \mathbb{1} \right\}$$

$$\mathbb{1}_{m} \triangleleft_{\mathbb{1}} \ \overset{\text{McL}}{\underset{283}{=}} \ \frac{\mathbb{1} \overset{\mathbb{L}}{\leftarrow} \mathbb{1}^{m} \ \text{m-lin}}{\mathbb{L} \underbrace{\mathbb{1}^{1} \cdot\cdot e \cdot\cdot \mathbb{1}^{m}} = 0}$$

$$\mathbb{1}_{\mathbb{N}} \triangleleft_{\mathbb{1}} = \sum_{m} \mathbb{1}_{m} \triangleleft_{\mathbb{1}}$$

$$\overbrace{\mathbb{L} \underbrace{\mathbb{1}_{m} \triangleleft_{\mathbb{1}}}} \mathbb{1}^{1} \overset{\cdots}{\otimes} \mathbb{1}^{m} = \overbrace{\mathbb{L} \underbrace{\mathbb{1} \mathbb{1}^{1}} \overset{\cdots}{\otimes} \underbrace{\mathbb{1} \mathbb{1}^{m}}} \ \mathbb{1}$$

$$\mathbb{1}_{m} \triangleleft_{\mathbb{1}} \overset{d}{\leftarrow} \mathbb{1}_{m+1} \triangleleft_{\mathbb{1}}$$

$$\underbrace{\mathbb{L} d} \underbrace{\mathbb{1}^{0} \overset{\cdots}{\otimes} \mathbb{1}^{m}} = \mathbb{1}^{0} \ltimes \overbrace{\mathbb{L} \underbrace{\mathbb{1}^{1} \overset{\cdots}{\otimes} \mathbb{1}^{m}}} - \overset{m}{(-1)} \overbrace{\mathbb{L} \underbrace{\mathbb{1}^{0} \overset{\cdots}{\otimes} \mathbb{1}^{m-1}}} \rtimes \mathbb{1}^{m} - \sum_{j}^{m} \overset{j}{(-1)} \mathbb{L} \overbrace{\underbrace{\mathbb{1}^{0} \overset{\cdots}{\otimes} \mathbb{1}^{j-1}} \otimes \underbrace{\mathbb{1}^{j} \times \mathbb{1}^{j+1}} \otimes \underbrace{\mathbb{1}^{j+2} \overset{\cdots}{\otimes} \mathbb{1}^{m}}}$$

$$= \mathbb{1}^{0} \ltimes \overbrace{\mathbb{L} \underbrace{\mathbb{1}^{1} \overset{\cdots}{\otimes} \mathbb{1}^{m}}} + \overset{m+1}{(-1)} \overbrace{\mathbb{L} \underbrace{\mathbb{1}^{0} \overset{\cdots}{\otimes} \mathbb{1}^{m-1}}} \rtimes \mathbb{1}^{m} + \sum_{1 \leqslant i \leqslant m} \overset{i}{(-1)} \mathbb{L} \overbrace{\mathbb{1}^{0} \overset{\cdots}{\otimes} \mathbb{1}^{i-2} \otimes \underbrace{\mathbb{1}^{i-1} \times \mathbb{1}^{i}} \otimes \mathbb{1}^{i+1} \overset{\cdots}{\otimes} \mathbb{1}^{m}}$$

$$\underbrace{\mathbb{L} \mathbb{1}} d = \underbrace{\mathbb{L} d} \mathbb{1}$$

$$\mathbb{L} d d = 0$$

$$\tilde{\mathbb{1}} = \mathbb{1} \triangleleft_{\mathbb{1}} \mathbb{1} \text{lin} \mathbb{1}$$

$$\underbrace{\mathbb{1} \tilde{\ltimes} \mathbb{L}} \mathbb{1} = \mathbb{1} \ltimes \underbrace{\mathbb{L} \mathbb{1}}$$

$$\underbrace{\mathbb{L} \tilde{\rtimes} \mathbb{1}} \mathbb{1} = \mathbb{L} \underbrace{\mathbb{1} \times \mathbb{1}} - \underbrace{\mathbb{L} \mathbb{1}} \rtimes \mathbb{1}$$

$$\mathbb{1}_{m+1} \triangleleft_{\mathbb{1}} \xleftarrow[\exists]{\varphi_m} \tilde{\mathbb{1}}_{m} \triangleleft_{\mathbb{1}}$$

$$d_m \varphi_m = \varphi_{m-1} \tilde{d}_{m-1} \Longrightarrow d_m d_{m+1} \varphi_{m+1} = d_m \varphi_m \tilde{d}_m = \varphi_{m-1} \tilde{d}_{m-1} \tilde{d}_m \underset{\text{ind}}{=} 0$$