

$$\mathbb{C} \begin{array}{c} \triangleleft \\ 0 \end{array} \mathbb{1} = \mathcal{U} | \mathbb{1} \begin{array}{c} \triangleleft \\ 0 \end{array} \mathbb{1} = \frac{\mathcal{U} | \mathbb{1} \xleftarrow{\alpha} \mathbb{1}}{\begin{array}{c} \text{* - hom} \\ \mathbb{1} \xrightarrow[\alpha \text{ fax}]{\text{abg}} \mathbb{1} \curvearrowright \mathbb{1} = \begin{cases} 0 \\ \mathbb{1} \end{cases} \end{array}}$$

$$\alpha \text{ irr} \Leftrightarrow \alpha \mathbb{1}' = \frac{\mathbb{1} \in \mathcal{U} | \mathbb{1}}{\bigwedge_{\mathbb{1}} \mathbb{1} \xrightarrow{\alpha} \mathbb{1} = \mathbb{1} \mathbb{1}} = \mathbb{C} I$$

$$\Rightarrow: \mathbb{1} \in \alpha \mathbb{1}' \Rightarrow \mathbb{1}^* \xrightarrow{\alpha} \mathbb{1} = \overline{\mathbb{1} \mathbb{1}'}^* = \overline{\mathbb{1} \alpha \mathbb{1}'}^* = \mathbb{1} \mathbb{1}'^* \Rightarrow \mathbb{1}^* \in \alpha \mathbb{1}' \xrightarrow{\text{OE}} \mathbb{1} = \mathbb{1}^*$$

$$\xrightarrow{\text{SpecTh}} \bigvee_{E_\lambda} W^*(\mathbb{1}) \subset \alpha \mathbb{1}' \quad \mathbb{1} = \int_{d\lambda} E_\lambda \xrightarrow{\text{irred}} E_\lambda = \begin{cases} 0 \\ I \end{cases} \Rightarrow \mathbb{1} \in \mathbb{C} I$$

$$\Leftarrow: \mathbb{1} \xrightarrow[\alpha \text{ fax}]{\text{abg}} \mathbb{1} \Rightarrow \bigwedge_{\mathbb{1}} \bigwedge_{\mathbb{1}'} \mathbb{1} \mathbb{1}' = \overline{\mathbb{1} \mathbb{1}'} = 0 \Rightarrow \mathbb{1} \mathbb{1}' \in \mathbb{1} \alpha \text{ fax}$$

$$\Rightarrow \alpha \mathbb{1}' \ni \mathbb{1} \xrightarrow[\text{proj}]{P} \mathbb{1}' \Rightarrow P = \lambda I$$

$$\lambda^2 = \lambda \Rightarrow \begin{cases} \lambda = 0 \Rightarrow \mathbb{1}' = 0 \\ \lambda = 1 \Rightarrow \mathbb{1}' = \mathbb{1} \end{cases}$$

$\alpha: \mathbb{1}$ irred

$$\mathbb{1} \in \mathbb{1}$$

$$\mathbb{1} \mathbb{1} = 1 \Rightarrow \alpha: \mathbb{1}: \mathbb{1} \text{ cycl}$$

$$0 \neq \mathbb{1} \mathbb{1} \xrightarrow[\alpha \text{ fax}]{\text{abg}} \mathbb{1} \Rightarrow \mathbb{1} \mathbb{1}' = \mathbb{1}$$

cycl $\alpha: \mathbb{1} \text{ irr} \Leftrightarrow \alpha_{\mathbb{1}} \text{ pure}$

$$\Rightarrow : 0 \leq \mathbb{L} \leq \alpha_{\mathbb{1}} \xrightarrow{\text{LEM surj}} \bigvee_{\mathbb{L} \in \alpha_{\mathbb{1}}} \mathbb{L} = \mathbb{C}I$$

$$0 \leq \mathbb{L} \leq I$$

$$\mathbb{L} = \alpha_{\mathbb{1}} \Rightarrow \mathbb{L} = \lambda I$$

$$0 \leq \lambda \leq I \Rightarrow \mathbb{L} = \alpha_{\mathbb{1}} = \lambda^2 \alpha_{\mathbb{1}} \Rightarrow \alpha_{\mathbb{1}} \text{ pure}$$

$$\Leftarrow : \mathbb{1} \xrightarrow[\alpha \text{ fax}]{\text{abg}} \mathbb{1} \Rightarrow \alpha_{\mathbb{1}} \ni \mathbb{1} \xrightarrow[\text{proj}]{P} \mathbb{1} \Rightarrow \alpha_{\mathbb{1}} = \alpha_{P\mathbb{1}} + \alpha_{(I-P)\mathbb{1}} \geq \alpha_{P\mathbb{1}} \geq 0$$

$$\xrightarrow{\text{pure}} \alpha_{P\mathbb{1}} = \lambda^2 \alpha_{\mathbb{1}} = \alpha_{\lambda\mathbb{1}} \xrightarrow{\text{LEM inj}} P = \lambda I \Rightarrow \lambda = \begin{cases} 0 & \Rightarrow \mathbb{1} = 0 \\ 1 & \Rightarrow \mathbb{1} = \mathbb{1} \end{cases}$$