$$\mathbb{C}\,\triangledown_0\,\mathbb{1} = \frac{\mathbb{L} \in \mathbb{C}\,\triangledown_0\,\mathbb{1}}{\bigwedge_{\urcorner}^{\mathbb{1}} \mathbb{L}\urcorner^* \urcorner \geqslant 0}$$

$$\urcorner \bowtie_{\mathbb{L}} \urcorner' = \mathbb{L}\underbrace{\urcorner^* \urcorner'} \text{ pos semi-form}$$

$${}^{\mathbb{L}}\|\urcorner\| = \overbrace{\urcorner \bowtie_{\mathbb{L}} \urcorner}^{1/2} = \overbrace{\mathbb{L}\underbrace{\urcorner^* \urcorner}}^{1/2} \text{ semi-norm } \underset{\mathrm{CS}}{\Rightarrow} \overline{\|\mathbb{L}\underbrace{\urcorner^* \urcorner'}\|} = \overline{\|\urcorner \bowtie_{\mathbb{L}} \urcorner'\|} \leqslant {}^{\mathbb{L}}\|\urcorner\|\,{}^{\mathbb{L}}\|\urcorner'\| = \overbrace{\mathbb{L}\underbrace{\urcorner^* \urcorner}}^{1/2}\,\overbrace{\mathbb{L}\underbrace{\urcorner'^* \urcorner'}}^{1/2}$$

$$\mathbb{1} \text{ unit } \wedge \mathbb{L} \geqslant 0 \Rightarrow \begin{cases} \mathbb{L}\urcorner^* = \widehat{\mathbb{L}\urcorner}^* & \\ \mathbb{L} \text{ stet} & \|\mathbb{L}\| = \mathbb{L}e \end{cases}$$

$$\mathbb{L}\urcorner^* = \urcorner \bowtie_{\mathbb{L}} e = \widehat{e \bowtie_{\mathbb{L}} \urcorner}^* = \widehat{\mathbb{L}\urcorner}^*$$

$$\urcorner = \urcorner^* \in \mathbb{1}$$

$$\|\urcorner\| \leqslant 1 \underset{\mathrm{LEM}}{\Rightarrow} \bigvee_{\urcorner = \urcorner^*} e - \urcorner = \urcorner^2 \Rightarrow 0 \leqslant \mathbb{L}\underbrace{\urcorner^2} = \mathbb{L}\underbrace{e - \urcorner} = \mathbb{L}e - \mathbb{L}\urcorner \Rightarrow \mathbb{L}\urcorner \leqslant \mathbb{L}e$$

$$\|\urcorner\| \leqslant 1 \Rightarrow \|\mathbb{L}\urcorner\|^2 = \overline{\|e \bowtie_{\mathbb{L}} \urcorner\|} \underset{\mathrm{CS}}{\leqslant} e \bowtie_{\mathbb{L}} e\, \urcorner \bowtie_{\mathbb{L}} \urcorner = \widehat{\mathbb{L}e}\,\overbrace{\mathbb{L}\underbrace{\urcorner^* \urcorner}} \leqslant \widehat{\mathbb{L}e}^2 \Leftarrow \overline{\|\urcorner^* \urcorner\|} \leqslant \|\urcorner^*\|\|\urcorner\| = \|\urcorner\|^2 \leqslant 1$$

$$\Rightarrow \|\mathbb{L}\| = \bigvee_{\|\urcorner\| \leqslant 1} \|\mathbb{L}\urcorner\| \leqslant \mathbb{L}e \underset{\|e\| = 1}{\Rightarrow} \|\mathbb{L}\| = \mathbb{L}e$$

$$\mathbb{1} \text{ unit } \wedge \mathbb{L} \geqslant 0 \Rightarrow \|\mathbb{L}\urcorner\| \leqslant \|\mathbb{L}\|^{1/2}\,{}^{\mathbb{L}}\|\urcorner\|$$

$$\|\mathbb{L}\urcorner\| = \overline{\|e \bowtie_{\mathbb{L}} \urcorner\|} \underset{\mathrm{CS}}{\leqslant} {}^{\mathbb{L}}\|e\|\,{}^{\mathbb{L}}\|\urcorner\| = \overbrace{\mathbb{L}e}^{1/2}\,{}^{\mathbb{L}}\|\urcorner\| = \|\mathbb{L}\|^{1/2}\,{}^{\mathbb{L}}\|\urcorner\|$$

$$0 \leqslant \mathbb{L} \text{ stet}$$

$$\mathbb{L} \underbrace{\text{⅂}^*} = \overbrace{\mathbb{L}\text{⅂}}^*$$

$$\ulcorner \mathbb{L}\text{⅂} \urcorner \leqslant \ulcorner \mathbb{L} \urcorner^{1/2} \, {}^{\mathbb{L}}\ulcorner \text{⅂} \urcorner \Rightarrow \begin{cases} \tilde{\mathbb{L}}\,(\text{⅂}{:}a) := \mathbb{L}\text{⅂} + \ulcorner \mathbb{L} \urcorner a \in \mathbb{K}^{\triangledown}_{+} \, \mathbb{1} \boxtimes \mathbb{K}^{:\tilde{\mathbb{L}} | \mathbb{1} = \mathbb{L}} \\ \mathbb{1} \underset{\text{hull}}{\sqsubset} \mathbb{1} \boxtimes \mathbb{K} : \underset{\tilde{\mathbb{L}}}{\blacktriangledown} \end{cases}$$

$$\tilde{\mathbb{L}} \underbrace{(\text{⅂}{:}a)^* (\text{⅂}{:}a)} = \tilde{\mathbb{L}} \underbrace{\text{⅂}^* \text{⅂} + \text{⅂}^* a + \text{⅂} a^* + \ulcorner a \urcorner^2} = \mathbb{L} \underbrace{\text{⅂}^* \text{⅂}} + \underbrace{\mathbb{L}\text{⅂}^*} a + \underbrace{\mathbb{L}\text{⅂}} a^* + \ulcorner a \urcorner^2 \ulcorner \mathbb{L} \urcorner = {}^{\mathbb{L}}\ulcorner \text{⅂} \urcorner^2 + \overbrace{\mathbb{L}\text{⅂}}^* a + \underbrace{\mathbb{L}\text{⅂}} a^* + \ulcorner a \urcorner^2 \ulcorner \mathbb{L} \urcorner$$

$$\geqslant {}^{\mathbb{L}}\ulcorner \text{⅂} \urcorner^2 - 2 \ulcorner \mathbb{L}\text{⅂} \urcorner \ulcorner a \urcorner + \ulcorner a \urcorner^2 \ulcorner \mathbb{L} \urcorner \geqslant {}^{\mathbb{L}}\ulcorner \text{⅂} \urcorner^2 - 2 \ulcorner \mathbb{L} \urcorner^{1/2} \, {}^{\mathbb{L}}\ulcorner \text{⅂} \urcorner \ulcorner a \urcorner + \ulcorner a \urcorner^2 \ulcorner \mathbb{L} \urcorner = \overbrace{{}^{\mathbb{L}}\ulcorner \text{⅂} \urcorner - \ulcorner a \urcorner \ulcorner \mathbb{L} \urcorner^{1/2}}^2 \geqslant 0$$

$$\bigvee_{u_i \in \mathbb{1}_1 0} \mathbb{L}\, u_i \rightsquigarrow \ulcorner \mathbb{L} \urcorner \Rightarrow \ulcorner \mathbb{L} \urcorner \leftsquigarrow \mathbb{L}\, u_i \leqslant \ulcorner \mathbb{L} \urcorner^{1/2} \, {}^{\mathbb{L}}\ulcorner u_i \urcorner = \ulcorner \mathbb{L} \urcorner^{1/2} \overbrace{\mathbb{L} \underbrace{u_i^* u_i}}^{1/2} \underset{\ulcorner u_i^* u_i \urcorner \leqslant 1}{\leqslant} \ulcorner \mathbb{L} \urcorner^{1/2} \ulcorner \mathbb{L} \urcorner^{1/2} = \ulcorner \mathbb{L} \urcorner \Rightarrow {}^{\mathbb{L}}\ulcorner u_i \urcorner \rightsquigarrow \ulcorner \mathbb{L} \urcorner^{1/2}$$

$$\Rightarrow \ulcorner u_i - e \urcorner^2_{\tilde{\mathbb{L}}} = \tilde{\mathbb{L}} \underbrace{\overbrace{u_i - e}^* \overbrace{u_i - e}} = \mathbb{L}\left(u_i^* u_i\right) - \mathbb{L}\, u_i - \mathbb{L}\, u_i^* + \ulcorner \mathbb{L} \urcorner =$$

$${}^{\mathbb{L}}\ulcorner u_i \urcorner^2 - \mathbb{L}\, u_i - \underbrace{\mathbb{L} u_i}^* + \ulcorner \mathbb{L} \urcorner \rightsquigarrow \ulcorner \mathbb{L} \urcorner - \ulcorner \mathbb{L} \urcorner - \ulcorner \mathbb{L} \urcorner + \ulcorner \mathbb{L} \urcorner = 0 \Rightarrow \mathbb{1} \ni u_i \underset{\tilde{\mathbb{L}}}{\rightsquigarrow} e$$

$$\mathbb{1}\text{ a-unit } \mathbb{L}\in\mathbb{K}^{+}_{0\,\mathbb{1}}\text{ stet }\Rightarrow\begin{cases}\mathbb{L}\underbrace{\urcorner^{*}} = \overbrace{\mathbb{L}\urcorner}^{*}\\ \ulcorner\mathbb{L}\urcorner\urcorner \leqslant \ulcorner\mathbb{L}\urcorner^{1/2\,\mathbb{L}}\ulcorner\urcorner\urcorner \Rightarrow \bigvee\tilde{\mathbb{L}}\in\mathbb{K}^{+}_{0\,\mathbb{1}\times\mathbb{K}}\\ \mathbb{L}u_i \rightsquigarrow \ulcorner\mathbb{L}\urcorner\\ \mathbb{L}\underbrace{1^{*}\urcorner 1} \leqslant \mathbb{L}\underbrace{1^{*}1}\ulcorner\urcorner\urcorner\end{cases}$$

$$(1)\ \overbrace{\mathbb{L}\urcorner}^{*} \underset{\text{stet}}{\leftsquigarrow} \overbrace{\mathbb{L}\underbrace{u_i\urcorner}}^{*} = \overbrace{u_i\bowtie_{\mathbb{L}}\urcorner}^{*} = \urcorner\bowtie_{\mathbb{L}}u_i = \mathbb{L}\underbrace{\urcorner^{*}u_i} \underset{\text{stet}}{\rightsquigarrow} \mathbb{L}\underbrace{\urcorner^{*}}$$

$$(2)\ \ulcorner\mathbb{L}\urcorner\urcorner \leftsquigarrow \ulcorner\mathbb{L}\left(u_i\urcorner\right)\urcorner = \ulcorner u_i\bowtie_{\mathbb{L}}\urcorner\urcorner \underset{\text{CS}}{\leqslant} {}^{\mathbb{L}}\ulcorner u_i\urcorner\,{}^{\mathbb{L}}\ulcorner\urcorner\urcorner = \overbrace{\mathbb{L}\underbrace{u_i^2}}^{1/2}{}^{\mathbb{L}}\ulcorner\urcorner\urcorner \leqslant \ulcorner\mathbb{L}\urcorner^{1/2}\,{}^{\mathbb{L}}\ulcorner\urcorner\urcorner$$

$$(3)\ \bigwedge_{\urcorner\in\mathbb{1}} u_i\bowtie_{\tilde{\mathbb{L}}}\urcorner = \mathbb{L}\underbrace{u_i\urcorner} \rightsquigarrow \mathbb{L}\urcorner = e\bowtie_{\tilde{\mathbb{L}}}\urcorner$$

$$\mathbb{1}\underset{\text{hull}}{\sqsubset}\mathbb{1}\bowtie\mathbb{K}{:}\,\tilde{\mathbb{L}} \Rightarrow u_i \underset{\text{weak}}{\rightsquigarrow} e\in\mathbb{1}\times_{\mathbb{L}}\mathbb{K} \Rightarrow \ulcorner u_i - e\urcorner_{\tilde{\mathbb{L}}} \underset{\text{strong}}{\rightsquigarrow} 0 \underset{\tilde{\mathbb{L}}\text{ stet}}{\Rightarrow} \mathbb{L}u_i = \tilde{\mathbb{L}}u_i \rightsquigarrow \tilde{\mathbb{L}}e = \ulcorner\tilde{\mathbb{L}}\urcorner = \ulcorner\mathbb{L}\urcorner$$

$$(4)\ \mathbb{L}_1\urcorner = \mathbb{L}\underbrace{1^{*}\urcorner 1} \Rightarrow \mathbb{L}_1\underbrace{\urcorner^{*}\urcorner} = \mathbb{L}\underbrace{\overbrace{\urcorner 1}^{*}\overbrace{\urcorner 1}} \geqslant 0 \Rightarrow \mathbb{L}_1\in\mathbb{K}^{+}_{0\,\mathbb{1}} \Rightarrow \mathbb{L}_1\text{ stet } \Rightarrow \ulcorner\mathbb{L}_1\urcorner \leftsquigarrow \mathbb{L}_1 u_i = \mathbb{L}\underbrace{1^{*}u_i 1} \rightsquigarrow \mathbb{L}\underbrace{1^{*}1}$$

$$\mathbb{L}' \geqslant 0 \Rightarrow \ulcorner\mathbb{L}+\mathbb{L}'\urcorner = \ulcorner\mathbb{L}\urcorner + \ulcorner\mathbb{L}'\urcorner$$

$$\ulcorner\mathbb{L}+\mathbb{L}'\urcorner \leftsquigarrow \underbrace{\mathbb{L}+\mathbb{L}'}u_i = \mathbb{L}u_i + \mathbb{L}'u_i \rightsquigarrow \ulcorner\mathbb{L}\urcorner + \ulcorner\mathbb{L}'\urcorner$$

$$\mathbb{C} \xleftarrow[\text{lin}]{\mathbb{L}} \mathbb{1} \ \text{C*-alg} \ \mathbb{L} \geqslant 0 \Rightarrow \mathbb{L} \ \text{stet}$$

$$\bigvee_{\overline{⅂_i} \leqslant 1}^{⅂_i \geqslant 0} \mathbb{L}⅂_i \rightsquigarrow c := \curlyvee_{\overline{⅂} \leqslant 1}^{⅂ \geqslant 0} \mathbb{L}⅂$$

$$\bigwedge_{a_i}^{I \overset{1}{\triangleright} \mathbb{C}} \sum_i \overline{a_i ⅂_i} \leqslant \sum_i \overline{a_i} < \infty \underset{\mathbb{1}\ \text{voll}}{\Longrightarrow} \sum_i^{\mathbb{N}} \overline{a_i} ⅂_i \in \mathbb{1}$$

$$\bigwedge_i^{\mathbb{N}} \sum_i^n \overline{a_i} ⅂_i \in \mathbb{1}_+ \ \text{conv cone} \ \Rightarrow \overline{\sum_i^n a_i \mathbb{L} ⅂_i} \underset{\mathbb{L} \geqslant 0}{\leqslant} \sum_i^n \overline{a_i} \mathbb{L} ⅂_i = \mathbb{L} \sum_i^n \overline{a_i} ⅂_i \leqslant \mathbb{L} \sum_i^{\mathbb{N}} \overline{a_i} ⅂_i < \infty$$

$$\Rightarrow \mathbb{L} ⅂_i \in \overbrace{I \overset{1}{\triangleright} \mathbb{C}}^{\sharp} = I \overset{\infty}{\triangleright} \mathbb{C} \Rightarrow c < \infty$$

$$\begin{cases} ⅂ = ⅂^* \\ \overline{⅂} \leqslant 1 \end{cases} \Rightarrow \begin{cases} ⅂ = ⅂_+ - ⅂_- \\ \overline{⅂_\pm} \leqslant 1 \end{cases} \Rightarrow \overline{\mathbb{L}⅂} \leqslant \mathbb{L}⅂_+ + \mathbb{L}⅂_- \leqslant 2c$$

$$\overline{⅂} \leqslant 1 \Rightarrow \overline{\mathbb{L}⅂} \leqslant \overline{\mathbb{L} \frac{⅂ + ⅂^*}{2}} + \overline{\mathbb{L} \frac{⅂ - ⅂^*}{2i}} \leqslant 4c$$