$$\parallel \underset{\text{comm}}{\sim} \; \nparallel \Leftrightarrow \parallel = \underline{\parallel \wedge \nparallel} \vee \underline{\parallel \wedge \bar{\nparallel}}$$

$$\parallel \sim \; \nparallel \Leftrightarrow \parallel \sim \bar{\nparallel}$$

$$\parallel \sim \parallel$$

$$\parallel \sim \bar{\parallel}$$

$$\parallel \sim \; \nparallel \implies \nparallel \sim \parallel$$

$$\underline{\nparallel \wedge \parallel} \vee \underline{\nparallel \wedge \bar{\parallel}} = \underline{\nparallel \wedge \parallel} \vee \underline{\nparallel \wedge \overline{\underline{\parallel \wedge \nparallel} \vee \underline{\parallel \wedge \bar{\nparallel}}}} = \underline{\nparallel \wedge \parallel} \vee \overbrace{\nparallel \wedge \overline{\parallel \wedge \nparallel} \wedge \overline{\bar{\parallel} \vee \nparallel}}$$

$$= \underline{\parallel \wedge \nparallel} \vee \underbrace{\overbrace{\nparallel \wedge \overline{\bar{\parallel} \vee \nparallel}} \wedge \overline{\parallel \wedge \nparallel}}_{\overline{\text{Abs}}} \; \underline{\parallel \wedge \nparallel} \vee \underbrace{\nparallel \wedge \overline{\parallel \wedge \nparallel}}_{\parallel \wedge \nparallel \leqslant \nparallel} \underset{\text{oMod}}{=} \nparallel$$

$$\parallel \leqslant \; \nparallel \implies \parallel \sim \; \nparallel$$

$$\underline{\parallel \wedge \nparallel} \vee \underline{\parallel \wedge \bar{\nparallel}} = \parallel \vee \underline{\parallel \wedge \bar{\nparallel}} \underset{\overline{\text{Abs}}}{=} \parallel$$

$$\parallel \sim \; \nparallel \implies \parallel \wedge \; \nparallel = \parallel \wedge \underline{\nparallel \vee \bar{\parallel}}$$

$$\parallel \wedge \underline{\nparallel \vee \bar{\parallel}} \underset{\text{distr}}{\overset{\text{semi}}{\geqslant}} \underline{\parallel \wedge \nparallel} \vee \underline{\parallel \wedge \bar{\parallel}} = \underline{\parallel \wedge \nparallel} \vee o = \parallel \wedge \; \nparallel$$

$$\parallel \wedge \underline{\nparallel \vee \bar{\parallel}} \wedge \overline{\parallel \wedge \nparallel} = \parallel \wedge \underline{\parallel \wedge \bar{\nparallel}} \vee \underline{\parallel \wedge \nparallel} \underset{\parallel \sim \nparallel}{=} \parallel \wedge \bar{\parallel} = o$$

$$\underset{\text{oMod}}{\implies} \parallel \wedge \underline{\nparallel \vee \bar{\parallel}} = \parallel \wedge \; \nparallel$$

$$x \sim y \sim z \Longrightarrow \begin{cases} \underline{x \vee y} \wedge z &= \underline{x \wedge z} \vee \underline{y \wedge z} \\ \underline{y \vee x} \wedge z &= \underline{y \wedge z} \vee \underline{x \wedge z} \end{cases}$$

$$\underline{x \vee y} \wedge z \underset{\text{distr}}{\overset{\text{semi}}{\geqslant}} \underline{x \wedge z} \vee \underline{y \wedge z}$$

$$\overline{\underline{x \vee y} \wedge z} \wedge \overline{\underline{x \wedge z} \vee \underline{y \wedge z}} = \underline{x \vee y} \wedge z \wedge \overline{\underline{\bar{x} \vee \bar{z}}} \wedge \overline{\bar{y} \vee \bar{z}}$$

$$\underset{z \sim \bar{x}}{=} \underline{x \vee y} \wedge \underline{z \wedge \bar{x}} \wedge \underline{\bar{y} \vee \bar{z}} = \underline{x \vee y} \wedge \bar{x} \wedge \overline{z \wedge \underline{\bar{y} \vee \bar{z}}} \underset{z \sim \bar{y}}{=} \underline{x \vee y} \wedge \bar{x} \wedge z \wedge \bar{y}$$

$$= \underline{x \vee y} \wedge \overline{x \vee y} \wedge z = o \wedge z = o \Longrightarrow \underline{x \vee y} \wedge z = \underline{x \wedge z} \vee \underline{y \wedge z}$$

$$\underline{y \vee x} \wedge z \underset{\text{distr}}{\overset{\text{semi}}{\geqslant}} \underline{y \wedge z} \vee \underline{x \wedge z}$$

$$\underline{y \vee x} \wedge z \wedge \overline{\underline{y \wedge z} \vee \underline{x \wedge z}} = \underline{y \vee x} \wedge \overline{z \wedge \underline{\bar{y} \vee \bar{z}}} \wedge \underline{\bar{x} \vee \bar{z}} \underset{z \sim \bar{y}}{=} \underline{y \vee x} \wedge \underline{y \wedge \bar{z}} \wedge \underline{\bar{x} \vee \bar{z}}$$

$$= \overline{\underline{y \vee x} \wedge \bar{z}} \wedge y \wedge \underline{\bar{x} \vee \bar{z}} \underset{\bar{z} \sim x}{=} x \wedge \bar{z} \wedge y \wedge \underline{\bar{x} \vee \bar{z}} = \underline{x \wedge y} \wedge \overline{x \wedge y} \wedge \bar{z} = o \wedge \bar{z} = o$$

$$\Longrightarrow \underline{y \vee x} \wedge z = \underline{y \wedge z} \vee \underline{x \wedge z}$$

$$x \sim z \Longrightarrow x \frown z = \bar{x} \vee z$$

$$x \frown z = \underline{\bar{x} \wedge z} \vee \underline{\bar{x} \wedge \bar{z}} \vee \overline{x \wedge \underline{z \vee \bar{x}}} \underset{x \sim z}{=} \underline{\bar{x} \wedge z} \vee \underline{\bar{x} \wedge \bar{z}} \vee \underline{x \wedge z}$$

$$= \underline{z \wedge x} \vee \underline{z \wedge \bar{x}} \vee \underline{\bar{x} \wedge \bar{z}} \underset{z \sim x}{=} z \vee \underline{\bar{x} \wedge \bar{z}} = \overline{\bar{z} \wedge \underline{x \vee z}} \underset{\bar{z} \sim x}{=} \overline{\bar{z} \wedge x} = z \vee \bar{x}$$