$$\text{ortho } \mathbb{L} \text{ o-modular} \iff \mathbb{l} \leqslant \mathbb{k} \Longrightarrow \mathbb{l} \vee \underbrace{\mathbb{k} \wedge \bar{\mathbb{l}}}_{} = \mathbb{k} \iff \mathbb{l} \vee \overbrace{\bar{\mathbb{l}} \wedge \underbrace{\mathbb{l} \vee \mathbb{k}}_{}}^{} = \mathbb{l} \vee \mathbb{k}$$

$$\text{ortho } \mathbb{L} \text{ modular} \implies \mathbb{L} \text{ o-modular}$$

$$\mathbb{l} \curvearrowright \underbrace{\mathbb{l} \curvearrowright \mathbb{k}}_{} = \bar{\mathbb{l}} \vee \mathbb{k}$$

$$\mathbb{l} \curvearrowright \underbrace{\mathbb{l} \curvearrowright \mathbb{k}}_{} = \overbrace{\bar{\mathbb{l}} \wedge \underbrace{\mathbb{l} \curvearrowright \mathbb{k}}_{}}^{} \vee \underbrace{\bar{\mathbb{l}} \wedge \overline{\mathbb{l} \curvearrowright \mathbb{k}}}_{} \vee \underbrace{\mathbb{l} \wedge \overbrace{\bar{\mathbb{l}} \vee \underbrace{\mathbb{l} \curvearrowright \mathbb{k}}_{}}^{}}_{}$$

$$= \underbrace{\bar{\mathbb{l}} \wedge \overbrace{\bar{\mathbb{l}} \wedge \mathbb{k}}^{} \vee \bar{\mathbb{l}} \wedge \bar{\mathbb{k}} \vee \mathbb{l} \wedge \overbrace{\bar{\mathbb{l}} \vee \mathbb{k}}^{}}_{} \vee \underbrace{\bar{\mathbb{l}} \wedge \overbrace{\mathbb{l} \vee \bar{\mathbb{k}} \wedge \mathbb{l} \vee \mathbb{k} \wedge \bar{\mathbb{l}} \vee \overbrace{\mathbb{l} \wedge \bar{\mathbb{k}}}^{}}^{}}_{} \vee \underbrace{\mathbb{l} \wedge \overbrace{\bar{\mathbb{l}} \vee \bar{\mathbb{l}} \wedge \mathbb{k} \vee \bar{\mathbb{l}} \wedge \bar{\mathbb{k}} \vee \mathbb{l} \wedge \overbrace{\bar{\mathbb{l}} \vee \mathbb{k}}^{}}^{}}_{}$$

$$= \underbrace{\bar{\mathbb{l}} \wedge \bar{\mathbb{l}} \wedge \mathbb{k}}_{} \vee \underbrace{\bar{\mathbb{l}} \wedge \bar{\mathbb{l}} \wedge \bar{\mathbb{k}}}_{} \vee \overbrace{\bar{\mathbb{l}} \wedge \mathbb{l} \wedge \underbrace{\bar{\mathbb{l}} \vee \mathbb{k}}_{}}^{= o} \vee \overbrace{\underbrace{\mathbb{l} \vee \bar{\mathbb{k}}}_{} \wedge \underbrace{\mathbb{l} \vee \mathbb{k}}_{} \wedge \underbrace{\bar{\mathbb{l}} \wedge \bar{\mathbb{l}} \vee \overbrace{\bar{\mathbb{l}} \wedge \mathbb{l} \wedge \bar{\mathbb{k}}}^{= o}}_{}}^{}$$

$$\vee \underbrace{\mathbb{l} \wedge \bar{\mathbb{l}}}_{= o} \vee \underbrace{\mathbb{l} \wedge \bar{\mathbb{l}} \wedge \mathbb{k}}_{= o} \vee \underbrace{\mathbb{l} \wedge \bar{\mathbb{l}} \wedge \bar{\mathbb{k}}}_{= o} \vee \overbrace{\mathbb{l} \wedge \mathbb{l} \wedge \underbrace{\bar{\mathbb{l}} \vee \mathbb{k}}_{}}^{}$$

$$= \underbrace{\bar{\mathbb{l}} \wedge \mathbb{k}}_{} \vee \underbrace{\bar{\mathbb{l}} \wedge \bar{\mathbb{k}}}_{} \vee \overbrace{\underbrace{\mathbb{l} \vee \bar{\mathbb{k}}}_{} \wedge \underbrace{\mathbb{l} \vee \mathbb{k}}_{} \wedge \bar{\mathbb{l}}}^{} \vee \overbrace{\mathbb{l} \wedge \underbrace{\bar{\mathbb{l}} \vee \mathbb{k}}_{}}^{}$$

$$= \overbrace{\underbrace{\bar{\mathbb{l}} \wedge \mathbb{k}}_{} \vee \underbrace{\bar{\mathbb{l}} \wedge \bar{\mathbb{k}}}_{}}^{\leqslant \bar{\mathbb{l}}} \vee \overbrace{\overbrace{\underbrace{\bar{\mathbb{l}} \wedge \mathbb{k}}_{} \vee \underbrace{\bar{\mathbb{l}} \wedge \bar{\mathbb{k}}}_{}}^{} \wedge \bar{\mathbb{l}}}^{} \vee \overbrace{\mathbb{l} \wedge \underbrace{\bar{\mathbb{l}} \vee \mathbb{k}}_{}}^{} \underset{\text{oMod}}{=} \bar{\mathbb{l}} \vee \overbrace{\mathbb{l} \wedge \underbrace{\bar{\mathbb{l}} \vee \mathbb{k}}_{}}^{} \underset{\text{oMod}}{=} \bar{\mathbb{l}} \vee \mathbb{k}$$

$$\begin{cases} \mathbb{l} \geqslant \mathbb{k} \\ \mathbb{l} \wedge \bar{\mathbb{k}} = o \end{cases} \implies \mathbb{l} = \mathbb{k}$$

$$\mathbb{l} \underset{\text{oMod}}{=} \mathbb{k} \vee \underbrace{\mathbb{l} \wedge \bar{\mathbb{k}}}_{} = \mathbb{k} \vee o = \mathbb{k}$$

$$\mathbb{l} R \mathbb{k} = \underbrace{\mathbb{l} \wedge \mathbb{k}}_{} \vee \underbrace{\bar{\mathbb{l}} \wedge \bar{\mathbb{k}}}_{} = e \Longrightarrow \mathbb{l} = \mathbb{k}$$

$$\mathbb{l} \wedge \mathbb{k} = \underbrace{\mathbb{l} \wedge \mathbb{k}}_{} \vee o = \underbrace{\mathbb{l} \wedge \mathbb{k}}_{} \vee \bar{e} = \underbrace{\mathbb{l} \wedge \mathbb{k}}_{} \vee \overline{\underbrace{\mathbb{l} \wedge \mathbb{k}}_{} \vee \underbrace{\bar{\mathbb{l}} \wedge \bar{\mathbb{k}}}_{}} = \underbrace{\mathbb{l} \wedge \mathbb{k}}_{} \vee \overline{\overline{\mathbb{l} \wedge \mathbb{k}} \wedge \overline{\bar{\mathbb{l}} \wedge \bar{\mathbb{k}}}}$$

$$= \underbrace{\mathbb{l} \wedge \mathbb{k}}_{} \vee \overline{\overline{\mathbb{l} \wedge \mathbb{k}} \wedge \overbrace{\mathbb{l} \vee \mathbb{k}}^{}} \underset{\mathbb{l} \wedge \mathbb{k} \leqslant \mathbb{l} \vee \mathbb{k}}{\overset{\text{oMod}}{=}} \mathbb{l} \vee \mathbb{k}$$

$$\Longrightarrow \begin{cases} \mathbb{l} \\ \mathbb{k} \end{cases} \leqslant \mathbb{l} \vee \mathbb{k} = \mathbb{l} \wedge \mathbb{k} \leqslant \begin{cases} \mathbb{k} \\ \mathbb{l} \end{cases} \implies \mathbb{l} = \mathbb{k}$$

$$\begin{cases} \mathbb{1} \vee \mathbb{4} = e \\ \mathbb{1} \leqslant \bar{\mathbb{4}} \\ \mathbb{4} \leqslant \mathbb{4} \end{cases} \Longrightarrow \begin{cases} \mathbb{1} = \bar{\mathbb{4}} \\ \mathbb{4} = \mathbb{4} \end{cases}$$

$$\underbrace{\mathbb{4} \wedge \mathbb{4}} \vee \underbrace{\bar{\mathbb{4}} \wedge \bar{\mathbb{4}}} \underset{\bar{\mathbb{4}} \leqslant \bar{\mathbb{4}}}{\overset{\mathbb{4} \leqslant \mathbb{4}}{\leqq}} \mathbb{4} \vee \bar{\mathbb{4}} \underset{\mathbb{1} \leqslant \bar{\mathbb{4}} \leqslant \mathbb{4} \vee \bar{\mathbb{4}} \geqslant \mathbb{4}}{\geqslant} \mathbb{4} \vee \mathbb{1} = e \underset{\text{Lem}}{\Longrightarrow} \mathbb{4} = \mathbb{4}$$

analog $\mathbb{1} = \bar{\mathbb{4}}$

$$\mathbb{1} \curvearrowright \mathbb{4} = e \Longrightarrow \mathbb{1} \leqslant \mathbb{4}$$

$$\begin{cases} \underbrace{\bar{\mathbb{1}} \wedge \mathbb{4}} \vee \underbrace{\bar{\mathbb{1}} \wedge \bar{\mathbb{4}}} & \leqslant \bar{\mathbb{1}} \\ \mathbb{1} \wedge \underbrace{\bar{\mathbb{1}} \vee \mathbb{4}} & \leqslant \mathbb{1} \\ \overbrace{\underbrace{\bar{\mathbb{1}} \wedge \mathbb{4}} \vee \underbrace{\bar{\mathbb{1}} \wedge \bar{\mathbb{4}}} \vee \mathbb{1} \wedge \underbrace{\bar{\mathbb{1}} \vee \mathbb{4}}} & = \mathbb{1} \curvearrowright \mathbb{4} = e \end{cases} \underset{\text{Lem}}{\Longrightarrow} \begin{cases} \underbrace{\bar{\mathbb{1}} \wedge \mathbb{4}} \vee \underbrace{\bar{\mathbb{1}} \wedge \bar{\mathbb{4}}} & = \bar{\mathbb{1}} \\ \mathbb{1} \wedge \underbrace{\bar{\mathbb{1}} \vee \mathbb{4}} & = \mathbb{1} \end{cases}$$

$$\Longrightarrow \bar{\mathbb{1}} = \underbrace{\bar{\mathbb{1}} \wedge \mathbb{4}} \vee \underbrace{\bar{\mathbb{1}} \wedge \bar{\mathbb{4}}} \Longrightarrow \bar{\mathbb{1}} \sim \mathbb{4} \Longrightarrow \mathbb{1} \sim \mathbb{4} \Longrightarrow \mathbb{1} = \mathbb{1} \wedge \underbrace{\bar{\mathbb{1}} \vee \mathbb{4}} \underset{\text{distr}}{=} \underbrace{\underbrace{\mathbb{1} \wedge \bar{\mathbb{1}}}_{= o} \vee \underbrace{\mathbb{1} \wedge \mathbb{4}}} = \mathbb{1} \wedge \mathbb{4} \leqslant \mathbb{4}$$

$$\mathbb{1} \leqslant \mathbb{1} \vee \mathbb{4} \geqslant \mathbb{4} \Longrightarrow \mathbb{1} | \mathbb{1} \vee \mathbb{4} | \mathbb{4} \Longrightarrow \bar{\mathbb{1}} | \mathbb{1} \vee \mathbb{4} | \bar{\mathbb{4}}$$

analog $\bar{\mathbb{4}} | \mathbb{1} \vee \mathbb{4} \vee \mathbb{4} | \overline{\mathbb{1} \vee \mathbb{4}}$

$$\mathbb{1} \vee \mathbb{4} \geqslant \mathbb{4} \leqslant \mathbb{4} \vee \mathbb{4} \Longrightarrow \mathbb{1} \vee \mathbb{4} | \mathbb{4} | \mathbb{4} \vee \mathbb{4} \Longrightarrow \mathbb{1} \vee \mathbb{4} | \bar{\mathbb{4}} | \mathbb{4} \vee \mathbb{4}$$

$$\mathbb{1} \wedge \mathbb{4} \leqslant \mathbb{1} \leqslant \mathbb{1} \vee \mathbb{4} \geqslant \mathbb{4} \geqslant \mathbb{4} \wedge \mathbb{4} \Longrightarrow \underbrace{\mathbb{1} \wedge \mathbb{4}} \vee \underbrace{\bar{\mathbb{4}} \vee \bar{\mathbb{4}}} | \mathbb{1} \vee \mathbb{4} | \bar{\mathbb{1}} \vee \bar{\mathbb{4}}$$

$$\mathbb{1} \wedge \mathbb{4} \leqslant \mathbb{1} \vee \mathbb{4} \geqslant \mathbb{4} \wedge \mathbb{4} \Longrightarrow \mathbb{1} \wedge \mathbb{4} | \mathbb{1} \vee \mathbb{4} | \mathbb{4} \wedge \mathbb{4} \Longrightarrow \bar{\mathbb{1}} \vee \bar{\mathbb{4}} | \mathbb{1} \vee \mathbb{4} | \bar{\mathbb{4}} \vee \bar{\mathbb{4}}$$

$${}_0Q\colon \quad \underbrace{\mathbb{1} \vee \overbrace{\bar{\mathbb{1}} \wedge \underbrace{\mathbb{1} \vee \mathbb{4}}}} R \underbrace{\mathbb{1} \vee \mathbb{4}} = e$$

$$\text{LHS} \underset{\text{oMod}}{=} \text{RHS} \Longrightarrow \text{LHS } R \text{ RHS} = e$$

$$Q'_0:\quad \underbrace{\bar{\nparallel} \vee \text{⫲}} \curvearrowright \overbrace{\nparallel \curvearrowright \underbrace{\nparallel \curvearrowright \text{⫲}}} = e$$

$$\text{LHS} \ = \ \text{RHS} \ \Rightarrow \ \text{LHS} \ \curvearrowright \ \text{RHS} \ = e$$

$$Q11:\quad \overline{\nparallel R \text{⫲}} \vee \underbrace{\overline{\text{⫲} R \text{⋕}} \vee \overbrace{\nparallel R \text{⋕}}} = e$$

$$\overline{\nparallel R \text{⫲}} \vee \overline{\text{⫲} R \text{⋕}} \vee \underbrace{\nparallel R \text{⋕}} = \overbrace{\underbrace{\bar{\nparallel} \vee \bar{\text{⫲}}} \wedge \underbrace{\nparallel \vee \text{⫲}}}^{\bar{\nparallel}|\nparallel \vee \text{⫲}|\bar{\text{⫲}}} \vee \overbrace{\underbrace{\bar{\text{⫲}} \vee \bar{\text{⋕}}} \wedge \underbrace{\text{⫲} \vee \text{⋕}}}^{\bar{\text{⫲}}|\text{⫲} \vee \text{⋕}|\bar{\text{⋕}}} \vee \underbrace{\nparallel \wedge \text{⋕}} \vee \underbrace{\bar{\nparallel} \wedge \bar{\text{⋕}}}$$

$$= \overbrace{\bar{\nparallel} \wedge \underbrace{\nparallel \vee \text{⫲}}} \vee \overbrace{\bar{\text{⫲}} \wedge \underbrace{\nparallel \vee \text{⋕}}} \underset{\nparallel \vee \text{⋕}|\bar{\text{⫲}}|\text{⫲} \vee \text{⋕}}{\vee} \overbrace{\bar{\text{⫲}} \wedge \underbrace{\text{⫲} \vee \text{⋕}}} \vee \overbrace{\bar{\text{⋕}} \wedge \underbrace{\text{⫲} \vee \text{⋕}}} \vee \underbrace{\nparallel \wedge \text{⋕}} \vee \overline{\nparallel \vee \text{⋕}}$$

$$= \overbrace{\bar{\nparallel} \wedge \underbrace{\nparallel \vee \text{⫲}}} \vee \overbrace{\bar{\text{⫲}} \wedge \underbrace{\nparallel \vee \text{⫲} \vee \text{⋕}}} \underset{\bar{\text{⫲}}|\nparallel \vee \text{⫲} \vee \text{⋕}|\overline{\nparallel \vee \text{⋕}}}{\vee} \overline{\nparallel \vee \text{⋕}} \vee \overbrace{\bar{\text{⋕}} \wedge \underbrace{\text{⫲} \vee \text{⋕}}} \vee \underbrace{\nparallel \wedge \text{⋕}}$$

$$= \overbrace{\bar{\nparallel} \wedge \underbrace{\nparallel \vee \text{⫲}}} \vee \overbrace{\underbrace{\bar{\text{⫲}} \vee \overline{\nparallel \vee \text{⋕}}} \wedge \underbrace{\nparallel \vee \text{⫲} \vee \text{⋕} \vee \overline{\nparallel \vee \text{⋕}}}_{= \text{⫲} \vee e = e}} \vee \overbrace{\bar{\text{⋕}} \wedge \underbrace{\text{⫲} \vee \text{⋕}}} \vee \underbrace{\nparallel \wedge \text{⋕}}$$

$$= \overbrace{\bar{\nparallel} \wedge \underbrace{\nparallel \vee \text{⫲}}} \underset{\bar{\nparallel}|\nparallel \vee \text{⫲}|\bar{\text{⫲}}}{\vee} \bar{\text{⫲}} \vee \overline{\nparallel \vee \text{⋕}} \vee \overbrace{\bar{\text{⋕}} \wedge \underbrace{\text{⫲} \vee \text{⋕}}} \vee \underbrace{\nparallel \wedge \text{⋕}} = \overbrace{\underbrace{\bar{\nparallel} \vee \bar{\text{⫲}}} \wedge \underbrace{\nparallel \vee \text{⫲} \vee \bar{\text{⫲}}}_{= \nparallel \vee e = e}} \vee \overbrace{\bar{\text{⋕}} \wedge \underbrace{\text{⫲} \vee \text{⋕}}} \vee \underbrace{\nparallel \wedge \text{⋕}} \vee \overline{\nparallel \vee \text{⋕}}$$

$$= \bar{\nparallel} \vee \bar{\text{⫲}} \underset{\bar{\text{⫲}}|\text{⫲} \vee \text{⋕}|\bar{\text{⋕}}}{\vee} \overbrace{\bar{\text{⋕}} \wedge \underbrace{\text{⫲} \vee \text{⋕}}} \vee \underbrace{\nparallel \wedge \text{⋕}} \vee \overline{\nparallel \vee \text{⋕}} = \bar{\nparallel} \vee \overbrace{\underbrace{\bar{\text{⫲}} \vee \bar{\text{⋕}}} \wedge \underbrace{\bar{\text{⫲}} \vee \text{⫲} \vee \text{⋕}}_{= e \vee \text{⋕} = e}} \vee \overline{\nparallel \vee \text{⋕}} \vee \underbrace{\nparallel \wedge \text{⋕}}$$

$$= \bar{\nparallel} \vee \bar{\text{⫲}} \vee \bar{\text{⋕}} \vee \overline{\nparallel \vee \text{⋕}} \vee \underbrace{\nparallel \wedge \text{⋕}} = \bar{\text{⫲}} \vee \overline{\nparallel \vee \text{⋕}} \vee \overline{\nparallel \wedge \text{⋕}} \vee \underbrace{\nparallel \wedge \text{⋕}} = \bar{\text{⫲}} \vee \overline{\nparallel \vee \text{⋕}} \vee e = e$$

$$Q12: \quad \overline{\nparallel R \dagger} \vee \overbrace{\underbrace{\nparallel \wedge \yen} R \underbrace{\dagger \wedge \yen}} = e$$

$$\overbrace{\underbrace{\nparallel \wedge \yen} R \underbrace{\dagger \wedge \yen}} \vee \overline{\nparallel R \dagger} = \underbrace{\nparallel \wedge \dagger \wedge \yen} \vee \overbrace{\underbrace{\bar{\nparallel} \vee \bar{\yen}} \wedge \underbrace{\bar{\dagger} \vee \bar{\yen}}} \underset{\underbrace{\bar{\nparallel} \vee \bar{\yen}} \wedge \underbrace{\dagger - \vee \bar{\yen}} | \nparallel \vee \dagger | \bar{\nparallel} \vee \bar{\dagger}}{\vee} \overbrace{\underbrace{\bar{\nparallel} \vee \bar{\dagger}} \wedge \underbrace{\nparallel \vee \dagger}}$$

$$= \underbrace{\nparallel \wedge \dagger \wedge \yen} \vee \overbrace{\underbrace{\overbrace{\underbrace{\bar{\nparallel} \vee \bar{\yen}} \wedge \underbrace{\bar{\dagger} \vee \bar{\yen}}} \vee \bar{\nparallel} \vee \bar{\dagger}} \wedge \overbrace{\underbrace{\bar{\nparallel} \vee \bar{\yen}} \wedge \underbrace{\bar{\dagger} \vee \bar{\yen}}} \underset{\bar{\nparallel} \vee \bar{\yen} | \nparallel \vee \yen | \bar{\dagger} \vee \bar{\yen}}{\vee} \nparallel \vee \dagger}$$

$$= \underbrace{\nparallel \wedge \dagger \wedge \yen} \vee \overbrace{\underbrace{\overbrace{\underbrace{\bar{\nparallel} \vee \bar{\yen}} \wedge \underbrace{\bar{\dagger} \vee \bar{\yen}}} \vee \bar{\nparallel} \vee \bar{\dagger}} \wedge \underbrace{\bar{\nparallel} \vee \bar{\yen} \vee \nparallel \vee \dagger}_{= \bar{\yen} \vee \dagger \vee e = e} \wedge \underbrace{\bar{\dagger} \vee \bar{\yen} \vee \nparallel \vee \dagger}_{= \bar{\yen} \vee \nparallel \vee e = e}}$$

$$= \underbrace{\nparallel \wedge \dagger \wedge \yen} \vee \overbrace{\underbrace{\bar{\nparallel} \vee \bar{\yen}}_{\geqslant \bar{\yen}} \wedge \underbrace{\bar{\dagger} \vee \bar{\yen}}_{\geqslant \bar{\yen}}}^{\geqslant \bar{\yen}} \vee \bar{\nparallel} \vee \bar{\dagger} \geqslant \underbrace{\nparallel \wedge \dagger \wedge \yen} \vee \bar{\yen} \vee \bar{\nparallel} \vee \bar{\dagger} = e$$

$$Q13: \quad \overline{\nparallel R \dagger} \vee \underbrace{\bar{\nparallel} \vee \dagger} = e$$

$$\overline{\nparallel R \dagger} \vee \underbrace{\bar{\nparallel} \vee \dagger} = \overbrace{\underbrace{\bar{\nparallel} \vee \bar{\dagger}} \wedge \underbrace{\nparallel \vee \dagger}}^{\bar{\nparallel} | \nparallel \vee \dagger | \bar{\dagger}} \vee \bar{\nparallel} \vee \dagger = \overbrace{\bar{\nparallel} \wedge \underbrace{\nparallel \vee \dagger}} \vee \left(\bar{\dagger} \wedge \underbrace{\nparallel \vee \dagger} \right) \vee \bar{\nparallel} \vee \dagger$$

$$= \underbrace{\bar{\nparallel} \vee \overbrace{\bar{\nparallel} \wedge \underbrace{\nparallel \vee \dagger}}}_{\text{Abs}} \vee \underbrace{\dagger \vee \overbrace{\bar{\dagger} \wedge \underbrace{\nparallel \vee \dagger}}}_{\text{oMod}} = \bar{\nparallel} \vee \underbrace{\nparallel \vee \dagger} = e \vee \dagger = e$$