$$\mathbb{1} \text{ ortho} \iff \mathbb{1} \ni \mathbb{1} \curvearrowright \bar{\mathbb{1}} \in \mathbb{1}$$

$$\bar{\bar{\mathbb{1}}} = \mathbb{1}$$

$$\text{de Morgan} \begin{cases} \overline{\mathbb{1} \wedge \mathbb{4}} & = \bar{\mathbb{1}} \vee \bar{\mathbb{4}} \\ \overline{\mathbb{1} \vee \mathbb{4}} & = \bar{\mathbb{1}} \wedge \bar{\mathbb{4}} \end{cases}$$

$$\mathbb{1} \geqslant \mathbb{1} \wedge \mathbb{4} \leqslant \mathbb{4} \Longrightarrow \bar{\mathbb{1}} \leqslant \overline{\mathbb{1} \wedge \mathbb{4}} \geqslant \bar{\mathbb{4}} \Longrightarrow \overline{\mathbb{1} \wedge \mathbb{4}} \geqslant \bar{\mathbb{1}} \vee \bar{\mathbb{4}}$$

$$\bar{\mathbb{1}} \leqslant \mathbb{4} \geqslant \bar{\mathbb{4}} \Longrightarrow \mathbb{1} = \bar{\bar{\mathbb{1}}} \geqslant \bar{\mathbb{4}} \leqslant \bar{\bar{\mathbb{4}}} = \mathbb{4} \Longrightarrow \bar{\mathbb{4}} \leqslant \mathbb{1} \wedge \mathbb{4} \Longrightarrow \mathbb{4} = \bar{\bar{\mathbb{4}}} \geqslant \overline{\mathbb{1} \wedge \mathbb{4}} \Longrightarrow \overline{\mathbb{1} \wedge \mathbb{4}} = \bar{\mathbb{1}} \vee \bar{\mathbb{4}}$$

$$\Longrightarrow \overline{\mathbb{1} \vee \mathbb{4}} = \overline{\bar{\bar{\mathbb{1}}} \vee \bar{\bar{\mathbb{4}}}} = \overline{\overline{\bar{\mathbb{1}} \wedge \bar{\mathbb{4}}}} = \bar{\mathbb{1}} \wedge \bar{\mathbb{4}}$$

$$\mathbb{1} \geqslant \mathbb{4} \iff \bar{\mathbb{4}} \leqslant \bar{\mathbb{1}}$$

$$\Longrightarrow : \quad \mathbb{1} \leqslant \mathbb{4} \Longrightarrow \mathbb{1} = \mathbb{1} \wedge \mathbb{4} \Longrightarrow \bar{\mathbb{1}} = \overline{\mathbb{1} \wedge \mathbb{4}} = \bar{\mathbb{1}} \vee \bar{\mathbb{4}} \geqslant \bar{\mathbb{4}}$$

$$\Longleftarrow : \quad \bar{\mathbb{4}} \leqslant \bar{\mathbb{1}} \Longrightarrow \mathbb{1} = \bar{\bar{\mathbb{1}}} \leqslant \bar{\bar{\mathbb{4}}} = \mathbb{4}$$

$$\bar{\mathbb{1}} \wedge \mathbb{1} \underset{\text{null}}{=} \bar{\mathbb{1}} \wedge \mathbb{1} \wedge \mathbb{4} \iff \mathbb{1} \vee \bar{\mathbb{1}} \underset{\text{eins}}{=} \mathbb{1} \vee \bar{\mathbb{1}} \vee \mathbb{4}$$

$$\Longrightarrow : \quad \mathbb{1} \vee \bar{\mathbb{1}} = \bar{\bar{\mathbb{1}}} \vee \bar{\mathbb{1}} = \overline{\bar{\mathbb{1}} \wedge \mathbb{1}} \underset{\mathbb{4} = \bar{\mathbb{4}}}{=} \overline{\bar{\mathbb{1}} \wedge \mathbb{1} \wedge \bar{\mathbb{4}}} = \bar{\bar{\mathbb{1}}} \vee \bar{\mathbb{1}} \vee \bar{\bar{\mathbb{4}}} = \mathbb{1} \vee \bar{\mathbb{1}} \vee \mathbb{4}$$

$$\Longleftarrow : \quad \bar{\mathbb{1}} \wedge \mathbb{1} = \bar{\mathbb{1}} \wedge \bar{\mathbb{1}} = \overline{\mathbb{1} \vee \bar{\mathbb{1}}} \underset{\mathbb{4} = \bar{\mathbb{4}}}{=} \overline{\mathbb{1} \vee \bar{\mathbb{1}} \vee \bar{\mathbb{4}}} = \bar{\mathbb{1}} \wedge \bar{\bar{\mathbb{1}}} \wedge \bar{\bar{\mathbb{4}}} = \bar{\mathbb{1}} \wedge \mathbb{1} \wedge \mathbb{4}$$

$$\mathbb{1} \leqslant \mathbb{4} \Longrightarrow \bar{\mathbb{1}} \vee \mathbb{4} = e$$

$$\overline{\bar{\mathbb{1}} \vee \mathbb{4}} = \mathbb{1} \wedge \bar{\mathbb{4}} \leqslant \mathbb{4} \wedge \bar{\mathbb{4}} = o$$

$$\mathbb{1} \curvearrowright \mathbb{4} = \underbrace{\bar{\mathbb{1}} \wedge \mathbb{4}} \vee \underbrace{\bar{\mathbb{1}} \wedge \bar{\mathbb{4}}} \vee \overbrace{\mathbb{1} \wedge \underbrace{\mathbb{4} \vee \bar{\mathbb{1}}}} \leqslant \bar{\mathbb{1}} \vee \mathbb{4}$$

$$\nparallel \leqslant \nparallel \Longrightarrow \nparallel \curvearrowright \nparallel = e$$

$$\nparallel \leqslant \nparallel \leqslant \nparallel \vee \bar{\nparallel} \Longrightarrow \nparallel \wedge \underbrace{\nparallel \vee \bar{\nparallel}} = \nparallel$$

$$\bar{\nparallel} \leqslant \bar{\nparallel} \Longrightarrow \bar{\nparallel} \wedge \bar{\nparallel} = \bar{\nparallel}$$

$$\Longrightarrow \nparallel \curvearrowright \nparallel = \underbrace{\bar{\nparallel} \wedge \nparallel} \vee \bar{\nparallel} \vee \nparallel = \underbrace{\bar{\nparallel} \wedge \nparallel} \vee \overline{\nparallel \wedge \bar{\nparallel}} = \underbrace{\bar{\nparallel} \wedge \nparallel} \vee \overline{\bar{\nparallel} \wedge \nparallel} = e$$

$$\nparallel R \nparallel = \underbrace{\nparallel \wedge \nparallel} \vee \underbrace{\bar{\nparallel} \wedge \bar{\nparallel}} = \nparallel R \nparallel = \bar{\nparallel} R \bar{\nparallel}$$

$$\nparallel R \nparallel \underset{\text{Q1}}{=} e$$

$$\underbrace{\nparallel R \nparallel} R \underbrace{\nparallel R \nparallel} \underset{\text{Q2}}{=} e$$

$$\nparallel R \bar{\bar{\nparallel}} \underset{\text{Q3}}{=} e$$

$$\underbrace{\nparallel \wedge \nparallel} R \underbrace{\nparallel \wedge \nparallel} \underset{\text{Q4}}{=} e = \underbrace{\nparallel \vee \nparallel} R \underbrace{\nparallel \vee \nparallel}$$

$$\overline{\nparallel \vee \nparallel} R \underbrace{\bar{\nparallel} \wedge \bar{\nparallel}} \underset{\text{Q5}}{=} e$$

$$\overbrace{\nparallel \wedge \underbrace{\nparallel \vee \nparallel}} R \nparallel \underset{\text{Q6}}{=} e = \overbrace{\nparallel \vee \underbrace{\nparallel \wedge \nparallel}} R \nparallel$$

$$\overbrace{\nparallel \wedge \underbrace{\nparallel \vee \nparallel}} R \nparallel \underset{\text{Q7}}{=} e = \overbrace{\nparallel \vee \underbrace{\nparallel \wedge \nparallel}} R \nparallel$$

$$\underbrace{\bar{\nparallel} \wedge \nparallel} R \overbrace{\underbrace{\bar{\nparallel} \wedge \nparallel} \wedge \nparallel} \underset{\text{Q8}}{=} e$$

$$\overline{\nparallel R \nparallel} \vee \underbrace{\bar{\nparallel} R \bar{\nparallel}} \underset{\text{Q9}}{=} e$$

$$\overline{\nparallel \curvearrowright \nparallel} \vee \underbrace{\bar{\nparallel} \vee \nparallel} = e$$

$$Q1: \quad \nparallel R \nparallel = \underbrace{\nparallel \wedge \nparallel} \vee \underbrace{\bar{\nparallel} \wedge \bar{\nparallel}} = \nparallel \vee \bar{\nparallel} = e$$

$$\underline{Q}\, Q8: \quad \text{LHS} \;=\; \text{RHS} \;\Longrightarrow\; \text{LHS}\; R\; \text{RHS} \;= e$$

$$Q9: \quad \overline{\nparallel R \nparallel} \vee \underbrace{\bar{\nparallel} R \bar{\nparallel}} = \overline{\nparallel R \nparallel} \vee \underbrace{\nparallel R \nparallel} = e$$

$$\nparallel \curvearrowright \nparallel = \underbrace{\bar{\nparallel} \wedge \nparallel}_{\leqslant \bar{\nparallel}} \vee \underbrace{\bar{\nparallel} \wedge \bar{\nparallel}}_{\leqslant \bar{\nparallel}} \vee \overbrace{\nparallel \wedge \underbrace{\nparallel \vee \bar{\nparallel}}_{\leqslant \bar{\nparallel} \vee \nparallel}} \leqslant \bar{\nparallel} \vee \nparallel \underset{\text{Lem}}{\Longrightarrow} \overline{\nparallel \curvearrowright \nparallel} \vee \underbrace{\bar{\nparallel} \vee \nparallel} = e$$