$$\hbar \xrightarrow{\ \eta\ } \daleth \Rightarrow \hbar \xrightarrow{\ \overline{\eta}\ } \mathbb{R}_+$$

$$\overline{\overline{\eta}} = |\overline{\eta}|$$

$$1 \in {}^{\hbar}\!\!\lrcorner_0^0\, \daleth \Rightarrow \overline{\overline{1}} \in {}^{\hbar}\!\!\lrcorner_0^0\, \mathbb{R}_+ \overline{\overline{1}} = \int_{\mu}^{\hbar} \overline{\overline{1}} < \infty$$

$$\overline{\overline{\eta + \acute{\eta}}} \leqslant \overline{\overline{\eta}} + \overline{\overline{\acute{\eta}}}$$

$$\overline{\overline{\eta a}} = \overline{\overline{\eta}}\,\overline{a}$$

$$\overline{\eta + \acute{\eta}} \leqslant \overline{\eta} + \overline{\acute{\eta}} \Rightarrow \overline{\overline{\eta + \acute{\eta}}} = |\overline{\eta + \acute{\eta}}| \leqslant |\overline{\eta}| + |\overline{\acute{\eta}}| = \overline{\overline{\eta}} + \overline{\overline{\acute{\eta}}}$$

$$\overline{\eta a} = \overline{\eta}\,\overline{a} \Rightarrow \overline{\overline{\eta a}} = |\overline{\eta a}| = |\overline{\eta}\,\overline{a}| = |\overline{\eta}|\overline{a} = \overline{\overline{\eta}}\,\overline{a}$$

$$\hbar \xrightarrow[\eta_k]{\ \eta\ } \daleth \text{ voll}$$

$$\bigwedge_{x \in \hbar} \daleth \ni {}^{x}\eta_k \rightsquigarrow\Rightarrow {}^{x}\eta_k \rightsquigarrow {}^{x}\eta \text{ denn } \bigwedge_{0 \leqslant k} \begin{cases} \overline{\eta - \eta_k} \leqslant \sum_{j \geqslant k} \overline{\eta_{j+1} - \eta_j} \\ \overline{\overline{\eta - \eta_k}} \leqslant \sum_{j \geqslant k} \overline{\overline{\eta_{j+1} - \eta_j}} \end{cases}$$

$$\bigwedge_{x \in \text{ħ}} \overline{{}^{x}\text{ɿ} - {}^{x}\text{ɿ}_k} \leqslant \sum_{j \geqslant k} \overline{{}^{x}\text{ɿ}_{j+1} - {}^{x}\text{ɿ}_j}$$

$$\sum_{j \geqslant k} \overline{{}^{x}\text{ɿ}_{j+1} - {}^{x}\text{ɿ}_j} \underset{\text{OE}}{<} \infty$$

$$\underset{\text{⅂ voll}}{\Longrightarrow} \text{⅂} \ni \sum_{j \geqslant k} \overline{{}^{x}\text{ɿ}_{j+1} - {}^{x}\text{ɿ}_j} \leftsquigarrow \sum_{k \leqslant j \leqslant \ell} \overline{{}^{x}\text{ɿ}_{j+1} - {}^{x}\text{ɿ}_j} = {}^{x}\text{ɿ}_{\ell+1} - {}^{x}\text{ɿ}_\ell \rightsquigarrow {}^{x}\text{ɿ} - {}^{x}\text{ɿ}_k \Longrightarrow$$

$${}^{x}\text{ɿ} - {}^{x}\text{ɿ}_k = \sum_{j \geqslant k} \overline{{}^{x}\text{ɿ}_{j+1} - {}^{x}\text{ɿ}_j} \Longrightarrow \overline{{}^{x}\text{ɿ} - {}^{x}\text{ɿ}_k} \leqslant \sum_{j \geqslant k} \overline{{}^{x}\text{ɿ}_{j+1} - {}^{x}\text{ɿ}_j} \Longrightarrow (1)$$

$$\left\Vert \text{ɿ} - \text{ɿ}_k \right\Vert = \left| \overline{\text{ɿ} - \text{ɿ}_k} \right| \underset{\text{Lem}}{\leqslant} \sum_{j \geqslant k} \left| \overline{\text{ɿ}_{j+1} - \text{ɿ}_j} \right| = \sum_{j \geqslant k} \left\Vert \text{ɿ}_{j+1} - \text{ɿ}_j \right\Vert \Longrightarrow (2)$$

$${}^{\text{ħ}}\triangle^{1}_{m}\text{⅂} = \frac{\text{ħ} \xrightarrow{\text{ɿ}} \text{⅂}}{\bigvee \text{ɿ}_k \in {}^{\text{ħ}}\triangle^{0}_{0}\text{⅂} : \quad \left\Vert \text{ɿ} - \text{ɿ}_k \right\Vert \rightsquigarrow 0}$$

$${}^{\text{ħ}}\triangle^{1}_{m}\text{⅂} \ni \text{ɿ} \Rightarrow \left\Vert \text{ɿ} \right\Vert < \infty$$

$$\daleth \overset{\int_\mu^\hbar}{\longleftarrow} \hbar_m^1 \triangle \daleth : \int_\mu^\hbar \gimel := \lim \int_\mu^\hbar \gimel_k \begin{cases} \gimel_k \in \hbar_0^0 \triangle \daleth \\ \overline{\gimel - \gimel_k} \rightsquigarrow 0 \end{cases}$$

$$\begin{cases} \gimel_k \in \hbar_0^0 \triangle \daleth \\ \overline{\gimel - \gimel_k} \rightsquigarrow 0 \end{cases} \Rightarrow \overline{\int_\mu^\hbar \gimel_k - \int_\mu^\hbar \gimel_\ell} = \overline{\int_\mu^\hbar \underbrace{\gimel_k - \gimel_\ell}} \leqslant \int_\mu^\hbar \overline{\gimel_k - \gimel_\ell} = \overline{\gimel_k - \gimel_\ell} \leqslant \overline{\gimel_k - \gimel} + \overline{\gimel - \gimel_\ell} \rightsquigarrow 0$$

$$\Rightarrow \daleth \ni \int_\mu^\hbar \gimel_k \rightsquigarrow \underset{\text{Ex}}{\Rightarrow} \lim \int_\mu^\hbar \gimel_k$$

$$\begin{cases} \dot\gimel_k \in \hbar_0^0 \triangle \daleth \\ \overline{\gimel - \dot\gimel_k} \rightsquigarrow 0 \end{cases} \Rightarrow \overline{\int_\mu^\hbar \gimel_k - \int_\mu^\hbar \acute\gimel_k} = \overline{\int_\mu^\hbar \underbrace{\gimel_k - \acute\gimel_k}} \leqslant \int_\mu^\hbar \overline{\gimel_k - \acute\gimel_k} = \overline{\gimel_k - \acute\gimel_k} \leqslant \overline{\gimel_k - \gimel} + \overline{\gimel - \acute\gimel_k} \rightsquigarrow 0$$

$$\underset{\text{Eind}}{\Rightarrow} \lim \int_\mu^\hbar \gimel_k = \lim \int_\mu^\hbar \acute\gimel_k$$

$$\hbar_m^1 \triangle \daleth \in \nabla_0^n \mathbb{K} \text{ halb-norm } \Rightarrow \hbar_m^1 \triangle \daleth \ulcorner \frac{1 \in \hbar_m^1 \triangle \daleth}{\overline{1} = 0} \in \nabla_0^n \mathbb{K} \text{ norm}$$

$$\dot\gimel \in \hbar_m^1 \triangle \daleth \Rightarrow \begin{cases} \bigvee \dot\gimel_k \in \hbar_0^0 \triangle \daleth \\ \overline{\dot\gimel - \dot\gimel_k} \rightsquigarrow 0 \end{cases} \Rightarrow \gimel_k a + \gimel_k \acute a \in \hbar_0^0 \triangle \daleth$$

$$\overline{\underbrace{\gimel a + \acute\gimel \acute a} - \underbrace{\gimel_k a + \acute\gimel_k \acute a}} = \overline{\underbrace{\gimel - \gimel_k} a + \underbrace{\acute\gimel - \acute\gimel_k} \acute a} \leqslant \overline{\gimel - \gimel_k}\,\overline{a} + \overline{\acute\gimel - \acute\gimel_k}\,\overline{\acute a} \rightsquigarrow 0 \Rightarrow \gimel a + \acute\gimel \acute a \in \hbar_m^1 \triangle \daleth$$

$$\int_\mu^\hbar \underbrace{\gimel a + \acute\gimel \acute a} \backsim \int_\mu^\hbar \underbrace{\gimel_k a + \acute\gimel_k \acute a} = \underbrace{\int_\mu^\hbar} \gimel_k a + \underbrace{\int_\mu^\hbar} \acute\gimel_k \acute a \rightsquigarrow \underbrace{\int_\mu^\hbar} \gimel a + \underbrace{\int_\mu^\hbar} \acute\gimel \acute a$$

$$\mathtt{l} \xleftarrow[\text{stet}]{\int\limits_{\mu}^{\hbar}} {}^{\hbar}\triangle^{1}_{m}\,\mathtt{l} \ni \eta \Rightarrow \begin{cases} \lceil\eta\rceil \in {}^{\hbar}\triangle^{1}_{m}\mathbb{R}_{+} \\ \overline{\int\limits_{\mu}^{\hbar} \eta} \leqslant \int\limits_{\mu}^{\hbar} \lceil\eta\rceil = \|\eta\| \end{cases}$$

$$\begin{cases} \eta_{k} \in {}^{\hbar}\triangle^{0}_{0}\,\mathtt{l} \\ \|\eta - \eta_{k}\| \rightsquigarrow 0 \end{cases} \Rightarrow \begin{cases} \lceil\eta_{k}\rceil \in {}^{\hbar}\triangle^{0}_{0}\mathbb{R}_{+} \\ \overline{\lceil\eta\rceil - \lceil\eta_{k}\rceil} \leqslant \lceil\eta - \eta_{k}\rceil \end{cases} \Rightarrow \overline{|\lceil\eta\rceil| - |\lceil\eta_{k}\rceil|} \leqslant \|\lceil\eta\rceil - \lceil\eta_{k}\rceil\| = |\lceil\eta\rceil - \lceil\eta_{k}\rceil| \leqslant |\lceil\eta - \eta_{k}\rceil| = \|\eta - \eta_{k}\| \rightsquigarrow 0$$

$$\Rightarrow \lceil\eta\rceil \in {}^{\hbar}\triangle^{1}_{m}\mathbb{R}_{+}$$

$$\overline{\int\limits_{\mu}^{\hbar} \eta} \leftsquigarrow \overline{\int\limits_{\mu}^{\hbar} \eta_{k}} \leqslant \int\limits_{\mu}^{\hbar} \lceil\eta_{k}\rceil \rightsquigarrow \int\limits_{\mu}^{\hbar} \lceil\eta\rceil \Rightarrow \overline{\int\limits_{\mu}^{\hbar} \eta} \leqslant \int\limits_{\mu}^{\hbar} \lceil\eta\rceil$$

$$\|\eta\| = |\lceil\eta\rceil| \leftsquigarrow |\lceil\eta_{k}\rceil| = \int\limits_{\mu}^{\hbar} |\eta_{k}| \rightsquigarrow \int\limits_{\mu}^{\hbar} \lceil\eta\rceil \Rightarrow \|\eta\| = \int\limits_{\mu}^{\hbar} \lceil\eta\rceil$$

$$\hbar\,{}_{0}^{1}\!\triangle\,\daleth \text{ voll } \Rightarrow \hbar\,{}_{m}^{1}\!\triangle\,\daleth \ulcorner \frac{\mathtt{1} \in \hbar\,{}_{m}^{1}\!\triangle\,\daleth}{\ulcorner\mathtt{1}\urcorner = 0} \text{ voll treu}$$

$$\hbar\,{}_{m}^{1}\!\triangle\,\daleth \ni \text{ʅ}_k \rightsquigarrow \underset{\text{OE}}{\Rightarrow} \overline{\text{ʅ}_k - \text{ʅ}_{k+1}} \leqslant 2^{-k} \Rightarrow \begin{cases} \bigvee \mathtt{1}_k \in \hbar\,{}_{0}^{0}\!\triangle\,\daleth \\ \overline{\text{ʅ}_k - \mathtt{1}_k} \leqslant 2^{-k} \end{cases}$$

$$\Rightarrow \overline{\mathtt{1}_k - \mathtt{1}_{k+1}} \leqslant \overline{\mathtt{1}_k - \text{ʅ}_k} + \overline{\text{ʅ}_k - \text{ʅ}_{k+1}} + \overline{\text{ʅ}_{k+1} - \mathtt{1}_{k+1}} \leqslant 2^{-k} + 2^{-k} + 2^{-1-k} \leqslant 2^{2-k}$$

$$\text{Sei } \hbar \xrightarrow{\mathtt{1}} \daleth \bigwedge_{x \in \hbar} \daleth \ni {}^{x}\mathtt{1}_k \rightsquigarrow\Rightarrow {}^{x}\mathtt{1} \leftsquigarrow {}^{x}\mathtt{1}_k \underset{\text{LEM}}{\Rightarrow} \overline{\mathtt{1} - \mathtt{1}_k} \leqslant \sum_{j \geqslant k} \overline{\mathtt{1}_{j+1} - \mathtt{1}_j}$$

$$\underset{\text{LEM}}{\Rightarrow} \overline{\mathtt{1} - \mathtt{1}_k} = |\overline{\mathtt{1} - \mathtt{1}_k}| \leqslant \sum_{j \geqslant k} |\overline{\mathtt{1}_{j+1} - \mathtt{1}_j}| \leqslant \sum_{j \geqslant k} \overline{\mathtt{1}_{j+1} - \mathtt{1}_j} \leqslant \sum_{j \geqslant k} 2^{2-j} = 2^{3-k} \Rightarrow \mathtt{1} \in \hbar\,{}_{m}^{1}\!\triangle\,\daleth$$

$$\overline{\text{ʅ}_k - \mathtt{1}} \leqslant \overline{\text{ʅ}_k - \mathtt{1}_k} + \overline{\mathtt{1}_k - \mathtt{1}} \leqslant 2^{-k} + 2^{3-k} \leqslant 2^{4-k} \rightsquigarrow 0 \Rightarrow \hbar\,{}_{0}^{1}\!\triangle\,\daleth \ni \text{ʅ}_k \rightsquigarrow \text{ʅ}: \hbar \to \daleth$$

$$\overline{\text{ʅ}_k} \leqslant \mathtt{1} \in \hbar\,{}_{0}^{1}\!\triangle\,\mathbb{R}_+ \underset{\text{DCT}}{\Rightarrow} \text{ʅ} \in \hbar\,{}_{0}^{1}\!\triangle\,\daleth \int_\mu^\hbar \text{ʅ}_k \rightsquigarrow \int_\mu^\hbar \text{ʅ}$$

$$\daleth = \mathbb{R}: -\mathtt{1} \leqslant \text{ʅ}_k \leqslant \mathtt{1}$$

$$\curlyvee_{k \leqslant j \leqslant \ell} \text{ʅ}_j \in \hbar\,{}_{0}^{1}\!\triangle\,\mathbb{R} \Rightarrow \hbar\,{}_{0}^{1}\!\triangle\,\mathbb{R}_+ \ni \mathtt{1} + \curlyvee_{k \leqslant j \leqslant \ell} \text{ʅ}_j \nearrow \mathtt{1} + \curlyvee_{k \leqslant j} \text{ʅ}_j \leqslant 2\mathtt{1} \underset{\text{MCT}}{\Rightarrow} \hbar\,{}_{0}^{1}\!\triangle\,\mathbb{R}_+ \ni \mathtt{1} + \curlyvee_{k \leqslant j} \text{ʅ}_j$$

$$\int_\mu^\hbar \underbrace{\mathtt{1} + \curlyvee_{k \leqslant j} \text{ʅ}_j} \nearrow \int_\mu^\hbar \underbrace{\mathtt{1} + \curlyvee_{k \leqslant j \leqslant \ell} \text{ʅ}_j}$$

$$\mathtt{1} \geqslant \curlyvee_{k \leqslant j} \text{ʅ}_j \nearrow \curlywedge_k \curlyvee_{j \geqslant k} \text{ʅ}_j = \limsup \text{ʅ}_j = \text{ʅ} \Rightarrow \hbar\,{}_{0}^{1}\!\triangle\,\mathbb{R}_+ \ni \mathtt{1} - \curlyvee_{j \geqslant k} \text{ʅ}_j \nearrow \mathtt{1} - \text{ʅ} \underset{\text{MCT}}{\Rightarrow} \mathtt{1} - \text{ʅ} \in \hbar\,{}_{0}^{1}\!\triangle\,\mathbb{R}_+ \Rightarrow \text{ʅ} \in \hbar\,{}_{0}^{1}\!\triangle\,\mathbb{R}$$

$$\int_\mu^\hbar \mathtt{1} - \int_\mu^\hbar \text{ʅ} = \int_\mu^\hbar \underbrace{\mathtt{1} - \text{ʅ}} \nearrow \int_\mu^\hbar \underbrace{\mathtt{1} - \curlyvee_{k \leqslant j} \text{ʅ}_j} = \int_\mu^\hbar \mathtt{1} - \int_\mu^\hbar \curlyvee_{k \leqslant j} \text{ʅ}_j \Rightarrow \int_\mu^\hbar \text{ʅ} \nwarrow \int_\mu^\hbar \curlyvee_{k \leqslant j} \text{ʅ}_j$$

$$\curlyvee_{k \leqslant j} \int_\mu^\hbar \text{ʅ}_j \leqslant \int_\mu^\hbar \curlyvee_{k \leqslant j} \text{ʅ}_j \text{ true for all k}$$

$$\Rightarrow \overline{\lim} \int_\mu^\hbar \text{ʅ}_j = \curlywedge_k \curlyvee_{j \geqslant k} \int_\mu^\hbar \text{ʅ}_j \leqslant \curlywedge_k \int_\mu^\hbar \curlyvee_{j \geqslant k} \text{ʅ}_j = \int_\mu^\hbar \text{ʅ} = -\int_\mu^\hbar \underbrace{-\text{ʅ}} \leqslant -\overline{\lim} \int_\mu^\hbar \underbrace{-\text{ʅ}_j} = \underline{\lim} \int_\mu^\hbar \text{ʅ}_j$$