

$$\text{mb} \quad \mathfrak{h} \triangle_m \bar{\mathbb{R}}_+ \xrightarrow[\mu]{\int \mathfrak{h}} \bar{\mathbb{R}}_+$$

$$\mathfrak{h} \triangle_m \bar{\mathbb{R}}_+ = \frac{\gamma \in \mathfrak{h} \triangle_m \bar{\mathbb{R}}_+}{\int_{\mu} \mathfrak{h} \gamma < \infty}$$

$$\left\{ \begin{array}{l} 1 = \sum_{a \in \mathfrak{h} 1} a \chi_{\mathfrak{h}^{-1} a} \\ a \in \mathbb{R}_+ : \quad \mathfrak{h}^{-1} a \in \mathcal{M} : \quad 1_{\mathfrak{h}} \text{ fin} \end{array} \right. \Rightarrow \int_{\mu} 1 = \sum_{a \in \mathfrak{h} 1} a \mathfrak{h}^{-1} a$$

$$\left\{ \begin{array}{l} 1 = \sum_{E \in \mathcal{E}} 1^E \chi_E \\ \mathfrak{h} = \bigcup_{E \in \mathcal{E}} E : \quad \text{fin } \mathcal{E} \subset \mathcal{M} : \quad 1^E \geq 0 \end{array} \right. \Rightarrow \int_{\mu} 1 = \sum_{E \in \mathcal{E}} 1^E \mathfrak{h}_E$$

$$\int_{\mu} \underline{1a + 1'a} = \overline{\int_{\mu} 1a} + \overline{\int_{\mu} 1'a}$$

$$\begin{aligned} 1 &= \sum_{\dot{E} \in \dot{\mathcal{E}}} \dot{1}^{\dot{E}} \chi_{\dot{E}} \Rightarrow 1a + 1'a = \sum_{E \in \mathcal{E}} \sum_{\dot{E} \in \dot{\mathcal{E}}} \underline{\dot{1}^E a + \dot{1}^{\dot{E}} \dot{a}} \chi_{E \cap \dot{E}} \Rightarrow \\ \int_{\mu} \underline{1a + 1'a} &= \sum_{E \in \mathcal{E}} \sum_{\dot{E} \in \dot{\mathcal{E}}} \underline{\dot{1}^E a + \dot{1}^{\dot{E}} \dot{a}} \mathfrak{h}_{E \cap \dot{E}} = \sum_{E \in \mathcal{E}} \sum_{\dot{E} \in \dot{\mathcal{E}}} \dot{1}^E \mathfrak{h}_{E \cap \dot{E}} a + \sum_{E \in \mathcal{E}} \sum_{\dot{E} \in \dot{\mathcal{E}}} \dot{1}^{\dot{E}} \mathfrak{h}_{E \cap \dot{E}} \dot{a} \\ &= \sum_{E \in \mathcal{E}} \dot{1}^E \sum_{\dot{E} \in \dot{\mathcal{E}}} \mathfrak{h}_{E \cap \dot{E}} a + \sum_{\dot{E} \in \dot{\mathcal{E}}} \dot{1}^{\dot{E}} \sum_{E \in \mathcal{E}} \mathfrak{h}_{E \cap \dot{E}} \dot{a} = \sum_{E \in \mathcal{E}} \dot{1}^E \mathfrak{h}_E a + \sum_{\dot{E} \in \dot{\mathcal{E}}} \dot{1}^{\dot{E}} \mathfrak{h}_{\dot{E}} \dot{a} = \overline{\int_{\mu} 1a} + \overline{\int_{\mu} 1'a} \end{aligned}$$

$$1\geqslant \acute{1}\Rightarrow \int\limits_{\mu}^{\mathfrak{h}}1\geqslant \int\limits_{\mu}^{\mathfrak{h}}\acute{1}\in \bar{\mathbb{R}}_+$$

$$0\leqslant 1-\acute{1}=\sum_{E\in \mathcal{E}}\sum_{\acute{E}\in \acute{\mathcal{E}}}\underbrace{\acute{1}^E-\acute{1}^{\acute{E}}}\chi_{E\cap \acute{E}}$$

$$\acute{1}^E<\acute{1}^{\acute{E}}\Rightarrow \downarrow_{E\cap \acute{E}}=0$$

$$\Rightarrow \int\limits_{\mu}^{\mathfrak{h}}1=\sum_{E\in \mathcal{E}}\sum_{\acute{E}\in \acute{\mathcal{E}}}\acute{1}^E\downarrow_{E\cap \acute{E}}=\sum_{\acute{1}^E\geqslant \acute{1}^{\acute{E}}}\acute{1}^E\downarrow_{E\cap \acute{E}}\geqslant \sum_{\acute{1}^E\geqslant \acute{1}^{\acute{E}}}\acute{1}^{\acute{E}}\downarrow_{E\cap \acute{E}}=\sum_{E\in \mathcal{E}}\sum_{\acute{E}\in \acute{\mathcal{E}}}\acute{1}^{\acute{E}}\acute{a}\downarrow_{E\cap \acute{E}}=\int\limits_{\mu}^{\mathfrak{h}}\acute{1}$$

$$\mathfrak{h}\overset{\gamma}{\underset{\text{mb}}{\longrightarrow}}\bar{\mathbb{R}}_+\Rightarrow \int\limits_{\mu}^{\mathfrak{h}}\gamma=\bigvee_{0\leqslant \acute{\gamma}\leqslant \gamma}^{\text{simple}}\int\limits_{\mu}^{\mathfrak{h}}\acute{1}\in \bar{\mathbb{R}}_+$$

$$0\leqslant \gamma\leqslant \acute{\gamma}\text{ meas}\Rightarrow \int\limits_{\mu}^{\mathfrak{h}}\gamma\leqslant \int\limits_{\mu}^{\mathfrak{h}}\acute{\gamma}$$

$$c\geqslant 0\Rightarrow \int\limits_{\mu}^{\mathfrak{h}}\gamma c=c\int\limits_{\mu}^{\mathfrak{h}}\gamma$$

$$0 \leq \gamma_n \rightsquigarrow \gamma \xRightarrow{\text{FAT}} \int \gamma \leq \lim_{\mu} \int_{\mu}^{\mathfrak{h}} \gamma_n$$

$$0 \leq 1 \leq \gamma$$

$$\text{if } \int_{\mu}^{\mathfrak{h}} 1 = \infty \Rightarrow \bigvee_{M \in \mathcal{M}} \left\{ \begin{array}{l} \downarrow_M^M = \infty \\ \uparrow_M^M > a > 0 \end{array} \right.$$

$$^M \gamma \geq ^M 1 > a \Rightarrow M_n := \bigcap_{k \geq n} \frac{\mathfrak{h}}{\gamma_k > a} \nearrow \bigcup_n M_n \supset M \Rightarrow \downarrow_{M_n} \nearrow \downarrow_M = \infty$$

$$\int_{\mu}^{\mathfrak{h}} \gamma_n \geq a \downarrow_{M_n} \Rightarrow \lim_{\mu} \int_{\mu}^{\mathfrak{h}} \gamma_n = \infty$$

$$\text{if } \int_{\mu}^{\mathfrak{h}} 1 < \infty \Rightarrow M = \frac{\mathfrak{h}}{1 > 0} \Rightarrow \downarrow_M < \infty$$

$$\bigwedge_{\varepsilon > 0} M_n := \bigcap_{k \geq n} \frac{\mathfrak{h}}{\gamma_k > (1 - \varepsilon) 1} \nearrow \bigcup_n M_n \supset M \Rightarrow M \perp M_n \nearrow 0 \Rightarrow \downarrow_{M \perp M_n} \nearrow 0 \Rightarrow \bigvee_m \bigwedge_{n \geq m} \downarrow_{M \perp M_n} \leq \varepsilon$$

$$\Rightarrow \int_{\mu}^{\mathfrak{h}} \gamma_n \geq \int_{\mathfrak{h}}^{M_n} \gamma_n \geq (1 - \varepsilon) \int_{\mathfrak{h}}^{M_n} 1 = (1 - \varepsilon) \overline{\int_{\mathfrak{h}}^M 1 - \int_{\mathfrak{h}}^{M \perp M_n} 1} \geq (1 - \varepsilon) \int_{\mathfrak{h}}^M 1 - \int_{\mathfrak{h}}^{M \perp M_n} 1 \geq \int_{\mathfrak{h}}^M 1 - \varepsilon \overline{\int_{\mathfrak{h}}^M 1 + \Upsilon M 1}$$

$$\xRightarrow[\varepsilon \nearrow 0]{} \int_{\mu}^{\mathfrak{h}} \gamma_n \geq \int_{\mathfrak{h}}^M 1 = \int_{\mu}^{\mathfrak{h}} 1 \Rightarrow \lim_{\mu} \int_{\mu}^{\mathfrak{h}} \gamma_n \geq \bigvee_{0 \leq 1 \leq \gamma} \int_{\mu}^{\mathfrak{h}} 1 = \int_{\mu}^{\mathfrak{h}} \gamma$$

$$0 \leq \mathfrak{I}_n \leq \mathfrak{I} \text{ mb}$$

$$\mathfrak{I}_n \underset{\text{ae}}{\rightsquigarrow} \mathfrak{I} \overset{\text{MCT}}{\implies} \int_{\mu}^{\mathfrak{h}} \mathfrak{I}_n \rightsquigarrow \int_{\mu}^{\mathfrak{h}} \mathfrak{I}$$

$$\mathfrak{I}_n \leq \mathfrak{I} \Rightarrow \int_{\mu}^{\mathfrak{h}} \mathfrak{I} \leq \lim_{\mu} \int_{\mu}^{\mathfrak{h}} \mathfrak{I}_n \leq \lim_{\mu}^{-} \int_{\mu}^{\mathfrak{h}} \mathfrak{I}_n \leq \int_{\mu}^{\mathfrak{h}} \mathfrak{I}$$

$$0 \leq \mathfrak{I} \text{ meas} : \acute{a} \geq 0 \Rightarrow \int_{\mu}^{\mathfrak{h}} \underbrace{\mathfrak{I}a + \mathfrak{I}\acute{a}} = a \int_{\mu}^{\mathfrak{h}} \mathfrak{I} + \acute{a} \int_{\mu}^{\mathfrak{h}} \mathfrak{I}$$

$$\begin{aligned} 0 \leq \mathfrak{I}_n \nearrow \mathfrak{I} \Rightarrow 0 \leq \mathfrak{I}_n a + \mathfrak{I}_n \acute{a} \nearrow \mathfrak{I}a + \mathfrak{I}\acute{a} \\ \Rightarrow a \int_{\mu}^{\mathfrak{h}} \mathfrak{I} + \acute{a} \int_{\mu}^{\mathfrak{h}} \mathfrak{I} \underset{\text{MCT}}{\rightsquigarrow} a \int_{\mu}^{\mathfrak{h}} \mathfrak{I}_n + \acute{a} \int_{\mu}^{\mathfrak{h}} \mathfrak{I}_n = \int_{\mu}^{\mathfrak{h}} \underbrace{\mathfrak{I}_n a + \mathfrak{I}_n \acute{a}} \underset{\text{MCT}}{\rightsquigarrow} \int_{\mu}^{\mathfrak{h}} \underbrace{\mathfrak{I}a + \mathfrak{I}\acute{a}} \end{aligned}$$

$$\mathfrak{I} \geq 0 \Rightarrow \int_{\mu}^{\mathfrak{h}} \mathfrak{I} = 0 \Rightarrow \mathfrak{I} \underset{\text{ae}}{=} 0$$

$$M_n = \mathfrak{I} \geqslant \frac{1}{2} (\mathfrak{h}) \Rightarrow \mathfrak{I} \geqslant \frac{1}{n} \chi_{M_n} \Rightarrow 0 = \int_{\mu}^{\mathfrak{h}} \mathfrak{I} \geqslant \int_{\mu}^{\mathfrak{h}} \frac{1}{n} \chi_{M_n} = \frac{1}{n} \mathfrak{I}_{M_n} \Rightarrow \mathfrak{I}_{\{\mathfrak{I} > 0\}} = \mathfrak{I}_{\bigcup_n M_n} \leqslant \sum_n \mathfrak{I}_{M_n} = 0 \Rightarrow \mathfrak{I} \underset{\text{ae}}{=} 0$$

$$0 \leq \mathfrak{I}_n \text{ meas} \Rightarrow \int_{\mu}^{\mathfrak{h}} \sum_{0 \leq n} \mathfrak{I}_n = \sum_{0 \leq n} \int_{\mu}^{\mathfrak{h}} \mathfrak{I}_n$$

$$\sum_{n \in N} \mathfrak{I}_n \nearrow \sum_{0 \leq n} \mathfrak{I}_n \Rightarrow \sum_{0 \leq n} \int_{\mu}^{\mathfrak{h}} \mathfrak{I}_n \nearrow \sum_{n \in N} \int_{\mu}^{\mathfrak{h}} \mathfrak{I}_n = \int_{\mu}^{\mathfrak{h}} \sum_{n \in N} \mathfrak{I}_n \overset{\text{MCT}}{\nearrow} \int_{\mu}^{\mathfrak{h}} \sum_{0 \leq n} \mathfrak{I}_n$$