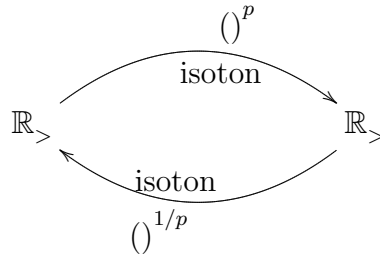
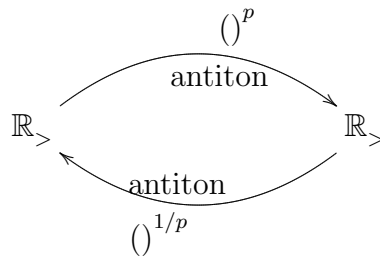


$$\mathbb{Z} \ni p > 0$$

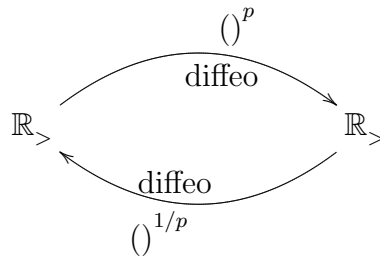


$$\frac{d}{dy} y^{1/n} = \overbrace{\frac{d}{dx} x^n}^{-1} = \overbrace{nx^{n-1}}^{-1} = \frac{x^{1-n}}{n} \underset{y=x^n}{=} = \frac{y^{(1-n)/n}}{n} = \frac{y^{1/n-1}}{n}$$

$$\mathbb{Z} \ni p < 0$$



$$\mathbb{Z} \ni p \neq 0$$



$$\frac{(\)^{1/p}}{p} = \frac{1}{p} (\)^{-1+1/p}$$

$$(x^{-1})^n = (x^n)^{-1} = x^{-n}$$

$$\mathbb{Z} \ni p:q: (x^p)^q = x^{pq} = (x^q)^p$$

$$r = \frac{a}{b} = \frac{c}{d} \Rightarrow (x^a)^{1/b} = (x^c)^{1/d} = x^r$$

$$\text{LHS}^{bc} = \left((x^a)^{1/b} \right)^{bc} = \left(\left((x^a)^{1/b} \right)^b \right)^c = (x^a)^c = x^{ac}$$

$$\text{RHS}^{da} = \left((x^c)^{1/d} \right)^{da} = \left(\left((x^c)^{1/d} \right)^d \right)^a = (x^c)^a = x^{ca}$$

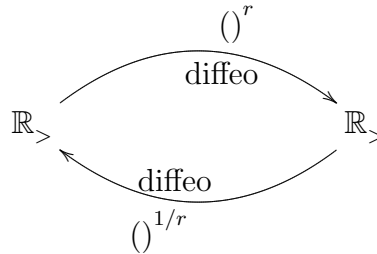
$$da = bc \xrightarrow{\text{inj}} \text{LHS} = \text{RHS}$$

$$(x^{1/q})^p = (x^p)^{1/q} = x^{p/q}$$

$$\text{LHS}^q = \left((x^{1/q})^p \right)^q = (x^{1/q})^{pq} = (x^{1/q})^{qp} = \left((x^{1/q})^q \right)^p = x^p = \text{RHS}^q \xrightarrow{\text{inj}} \text{LHS} = \text{RHS}$$

$$(x^{-1/q})^p = (x^{-p})^{1/q} = (x^{1/q})^{-p} = (x^p)^{-1/q} = x^{-p/q}$$

$$\mathbb{Q} \ni r \neq 0$$



$$\underline{()^{1/r}} = \frac{1}{r} ()^{-1+1/r}$$