

$$\mathbb{R}_> \xrightarrow[\text{bij}]{\mathcal{E}} \mathbb{R}$$

$${}^x \mathcal{E} = \frac{1}{x}$$

$$y \underline{\mathbf{e}} = y \mathbf{e}$$

$$1 = {}^x \mathcal{E} {}^x \mathcal{E} \underline{\mathbf{e}} = {}^x \mathcal{E} {}^x \mathcal{E} \mathbf{e} = {}^x \mathcal{E} x$$

$$|x| < 1 \Rightarrow {}^{1+x} \mathcal{E} = \sum_{n \geq 1} \binom{n-1}{-1} \frac{x^n}{n}$$

$$\partial_x {}^{1+x} \mathcal{E} = \frac{1}{1+x}$$

$$\partial_x \sum_{n \geq 1} \binom{n-1}{-1} \frac{x^n}{n} = \sum_{n \geq 1} \binom{n-1}{-1} n \frac{x^{n-1}}{n} = \sum_{n \geq 1} \binom{n-1}{-1} x^{n-1} = \sum_{n \geq 0} \binom{n}{-1} x^n = \sum_{n \geq 0} (-x)^n = \frac{1}{1-(-x)} = \frac{1}{1+x}$$

$${}^{1+0} \mathcal{E} = 0 = \sum_{n \geq 1} \binom{n-1}{-1} \frac{0^n}{n}$$

$$x = -1 \Rightarrow \sum_{n \geq 1} \binom{n-1}{-1} \frac{(-1)^n}{n} = - \sum_{n \geq 1} \frac{1}{n} = -\zeta_1(1) = -\infty$$

$$x = 1 \Rightarrow \sum_{n \geq 1} \binom{n-1}{-1} \frac{1^n}{n} = \sum_{n \geq 1} \binom{n-1}{-1} \frac{1}{n} = -\zeta_1(-1) = {}^2 \mathcal{E}$$