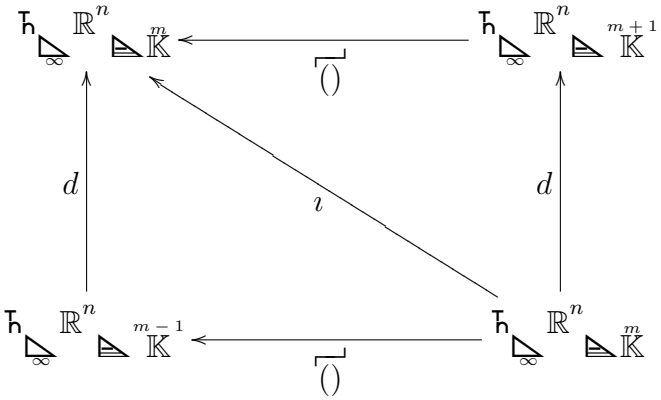
$\mathbb{R}^n \supset \mathbb{h}$ contractible

$$\mathbb{h}_{\infty}\mathbb{R}^n \mathbb{K}^{m-1} \xleftarrow[\text{htpy}]{\lceil () \rfloor} \mathbb{h}_{\infty}\mathbb{R}^n \mathbb{K}^{m}$$

$$\lceil \mathbb{h}^{m} \rfloor_{I} \mathbb{1} = \int_{dt}^{0|1} t^{m-1}\, t\mathbb{h}_{I}\mathbb{1} \sum_{i}^{I} \bigvee_{I\llcorner i}^{i} \mathbb{h}^{i}\, \mathbb{1}^{I\llcorner i}$$

$$\bigvee_{I}^{J} = (-1)^{I<J}$$



$$m > 0: \quad \mathbb{h}_{\infty}\mathbb{R}^n \mathbb{K}^{m} = 0$$

$$\mathbb{1} \in \mathbb{h}_{\infty}\mathbb{R}^n \mathbb{K}^{m} \Rightarrow d\mathbb{1} = 0 \Rightarrow \mathbb{1} = d\lceil \mathbb{1} \rfloor^{m} + \lceil d\mathbb{1} \rfloor^{m+1} = d\lceil \mathbb{1} \rfloor^{m} \in \mathbb{h}_{\infty}\mathbb{R}^n \mathbb{K}^{m}$$

$$\Rightarrow \mathbb{h}_{\infty}\mathbb{R}^n \mathbb{K}^{m} = \mathbb{h}_{\infty}\mathbb{R}^n \mathbb{K}^{m}$$

$$d\overset{m}{\omega} + \overset{m+1}{d\omega} = \omega$$

$$\overset{m+1}{\mathsf{h}\,d\omega} + d\,\overset{m}{\mathsf{h}\,\omega} = \mathsf{h}\overbrace{\sum_j \underbrace{{}_{j-I}\eta\,\omega}\,\eta^j \bowtie \eta^I}^{m+1} + d\int_{dt}^{0|1} t^{m-1}\, t\sim_I \omega \sum_i^I \overset{i}{\underset{I\llcorner i}{\vee}} \sim\eta^i\ \eta^{I\llcorner i}$$

$$= \mathsf{h}\overbrace{\sum_j^{n\llcorner I} \overset{j}{\underset{I}{\vee}}\eta^{I\cup j}\ \underbrace{{}_{j-I}\eta\,\omega}}^{m+1} + \int_{dt}^{0|1} t^{m-1} \sum_i^I \overset{i}{\underset{I\llcorner i}{\vee}} \sum_j \mathsf{h}\overbrace{{}_{j-}\eta \underbrace{t\sim_I \omega \sim \eta^i}}\ \eta^j \bowtie \eta^{I\llcorner i}$$

$$= \sum_j^{n\llcorner I} \overset{j}{\underset{I}{\vee}} \int_{dt}^{0|1} t^m\, t\mathsf{h}\overbrace{{}_{j-I}\eta\,\omega} \sum_k^{I\cup j} \overset{k}{\underset{I\cup j\llcorner k}{\vee}} \mathsf{h}^k\ \eta^{I\cup j\llcorner k} + \int_{dt}^{0|1} t^{m-1} \sum_i^I \overset{i}{\underset{I\llcorner i}{\vee}} \sum_{k\notin I\llcorner i} \overset{k}{\underset{I\llcorner i}{\vee}} \mathsf{h}\overbrace{{}_{k-}\eta \underbrace{t\sim_I \omega\sim\eta^i}}\ \eta^{I\llcorner i\cup k}$$

$$= \sum_j^{n\llcorner I} \overset{j}{\underset{I}{\vee}} \int_{dt}^{0|1} t^m\, t\mathsf{h}\overbrace{{}_{j-I}\eta\,\omega}\ \underbrace{\sum_i^I \overset{i}{\underset{I\cup j\llcorner i}{\vee}} \mathsf{h}^i \eta^{I\cup j\llcorner i} + \overset{j}{\underset{I}{\vee}} \mathsf{h}^j \eta^I} + \int_{dt}^{0|1} t^{m-1} \sum_i^I \overset{i}{\underset{I\llcorner i}{\vee}} \underbrace{\sum_j^{n\llcorner I} \overset{j}{\underset{I\llcorner i}{\vee}} t\, t\mathsf{h}\overbrace{{}_{j-I}\eta\,\omega}\mathsf{h}^i \eta^{I\llcorner i\cup j} + \overset{i}{\underset{I\llcorner i}{\vee}} t \underbrace{t\mathsf{h}\overbrace{{}_{i-I}\eta\,\omega}\mathsf{h}^i + t\mathsf{h}_I\omega}\ \eta^I}$$

$$= \sum_j^{n\llcorner I} \overset{j}{\underset{I}{\vee}} \int_{dt}^{0|1} t^m\, t\mathsf{h}\overbrace{{}_{j-I}\eta\,\omega}\ \overset{j}{\underset{I}{\vee}} \mathsf{h}^j\ \eta^I + \int_{dt}^{0|1} t^{m-1} \sum_i^I \overbrace{\overset{i}{\underset{I\llcorner i}{\vee}}\overset{i}{\underset{I\llcorner i}{\vee}}}^{=1} t\, t\mathsf{h}\overbrace{{}_{i-I}\eta\,\omega}\ \mathsf{h}^i + \int_{dt}^{0|1} t^{m-1} \sum_i^I \overbrace{\overset{i}{\underset{I\llcorner i}{\vee}}\overset{i}{\underset{I\llcorner i}{\vee}}}^{=1} t\mathsf{h}_I\omega\ \eta^I$$

$$+ \int_{dt}^{0|1} t^m \sum_i^I \sum_j^{n\llcorner I} \overbrace{\overset{i}{\underset{I\llcorner i}{\vee}}\overset{j}{\underset{I\llcorner i}{\vee}} + \overset{j}{\underset{I}{\vee}}\overset{i}{\underset{I\cup j\llcorner i}{\vee}}}^{=0}\ t\mathsf{h}\overbrace{{}_{j-I}\eta\,\omega}\ \mathsf{h}^i\ \eta^{I\llcorner i\cup j} = \sum_k \int_{dt}^{0|1} t^m\, t\mathsf{h}\overbrace{{}_{k-I}\eta\,\omega}\ \mathsf{h}^k\ \eta^I + m\int_{dt}^{0|1} t^{m-1}\, t\mathsf{h}_I\omega\ \eta^I$$

$$= \int_{dt}^{0|1} \underbrace{t^m \sum_k t\mathsf{h}\overbrace{{}_{k-I}\eta\,\omega}\mathsf{h}^k + m\, t^{m-1}\, t\mathsf{h}_I\omega}\ \eta^I = \int_{dt}^{0|1} \underbrace{t^m \frac{d}{dt}\, t\mathsf{h}_I\omega + m\, t^{m-1}\, t\mathsf{h}_I\omega}\ \eta^I = \int_{dt}^{0|1} \frac{d}{dt}\, t^m\, t\mathsf{h}_I\omega\ \eta^I = \boxed{t^m\, t\mathsf{h}_I\omega}_{t=0}^{t=1}\ \eta^I = \mathsf{h}_I\omega\ \eta^I = \mathsf{h}\omega$$

$$\overset{j}{\underset{I}{\vee}}\ \overset{i}{\underset{I\cup j\llcorner i}{\vee}} = \overset{j}{\underset{I\llcorner i}{\vee}}\ \overset{j}{\underset{i}{\vee}}\ \overset{i}{\underset{I\llcorner i}{\vee}}\ \overset{i}{\underset{j}{\vee}} = \overset{j}{\underset{i}{\vee}}\ \overset{i}{\underset{j}{\vee}}\ \overset{i}{\underset{I\llcorner i}{\vee}}\ \overset{j}{\underset{I\llcorner i}{\vee}} = -\overset{i}{\underset{I\llcorner i}{\vee}}\ \overset{j}{\underset{I\llcorner i}{\vee}}$$