

$$\bigvee^I_J = {^I>_1^J}$$

$$\begin{array}{ccc} \mathbb{h}_{\bigtriangleup_{\infty}} \underbrace{\mathbb{h}^{\pm}_{\bigtriangleup_{\mathbb{K}}^{\mathbb{N}}}} & \xrightarrow[\mathbb{h} \vDash]{} & \mathbb{h}_{\bigtriangleup_{\infty}} \underbrace{\mathbb{h}^{\pm}_{\bigtriangleup_{\mathbb{K}}^{\mathbb{N}}}} \\ & & \\ z(\mathbb{h} \vDash \mathbb{A}) & = & \mathbb{h}_z \vDash {}^z \mathbb{A} \end{array}$$

$$\mathbb{h} \bowtie = (\mathbb{h} \vDash) d + d (\mathbb{h} \vDash)$$

$$\begin{aligned} {}_M \mathbb{A}^z (\mathbb{h} \bowtie \mathbb{A}) &= {}_M \mathbb{A} \mathbb{h}_z {}^z \mathbb{A} + \sum_i^M \bigvee_{M \setminus i}^i \frac{{}^i \mathbb{A} \mathbb{h}_z}{{}^{M \setminus i} \mathbb{A}} {}^z \mathbb{A} \quad {}_M \mathbb{A} \sim (\mathbb{h} \bowtie \mathbb{A}) = \mathbb{h} \bowtie ({}_M \mathbb{A} \mathbb{A}) + \sum_i^M \bigvee_{M \setminus i}^i \frac{{}^i \mathbb{A} \times \mathbb{A}}{{}^{M \setminus i} \mathbb{A}} \mathbb{A} \\ {}_M \mathbb{A} \left(\mathbb{h} \vDash (d \mathbb{A}) + d (\mathbb{h} \vDash \mathbb{A}) \right) &= \frac{\mathbb{h}_z}{{}^M \mathbb{A}} {}^z (d \mathbb{A}) + \sum_i^M \bigvee_{M \setminus i}^i {}_{M \setminus i} \mathbb{A} \left({}_i \mathbb{A} {}^z \mathbb{h} \vDash \mathbb{A} \right) \\ &= {}_M \mathbb{A} \mathbb{h}_z {}^z \mathbb{A} - \sum_i^M \bigvee_{M \setminus i}^i \frac{\mathbb{h}_z}{{}^{M \setminus i} \mathbb{A}} {}^i \mathbb{A} {}^z \mathbb{A} + \sum_i^M \bigvee_{M \setminus i}^i \left(\frac{{}^i \mathbb{A} \mathbb{h}_z}{{}^{M \setminus i} \mathbb{A}} {}^z \mathbb{A} + \frac{\mathbb{h}_z}{{}^{M \setminus i} \mathbb{A}} {}^i \mathbb{A} {}^z \mathbb{A} \right) = {}_M \mathbb{A}^z (\mathbb{h} \bowtie \mathbb{A}) \\ {}_M \mathbb{A}_z {}^z (\mathbb{h} \bowtie \mathbb{A}) - \mathbb{h}_z {}^z \underline{{}^M \mathbb{A} \sim \mathbb{A}} &= {}_M \mathbb{A} \mathbb{h}_z {}^z \mathbb{A} + \sum_i^M \bigvee_{M \setminus i}^i \frac{{}^i \mathbb{A}_z \mathbb{h}_z}{{}^{M \setminus i} \mathbb{A}_z} {}^z \mathbb{A} - {}_M \mathbb{A} \mathbb{h}_z {}^z \mathbb{A} - \sum_i^M \bigvee_{M \setminus i}^i \frac{\mathbb{h}_z {}^i \mathbb{A}_z}{{}^{M \setminus i} \mathbb{A}_z} {}^z \mathbb{A} \\ &= \sum_i^M \bigvee_{M \setminus i}^i \frac{{}^i \mathbb{A}_z \mathbb{h}_z - \mathbb{h}_z {}^i \mathbb{A}_z}{{}^{M \setminus i} \mathbb{A}_z} {}^z \mathbb{A} \end{aligned}$$

$$(\mathbb{B} \bowtie) \times (\mathbb{B}' \bowtie) = (\mathbb{B} \times \mathbb{B}') \bowtie$$

$${}_M \mathbb{A} \mathbb{B} \mathbb{L}^z \underline{\mathbb{B} \bowtie \mathbb{A}} = {}_M \mathbb{A} \mathbb{B} \left(\mathbb{L} \underline{\mathbb{B} z} \underline{\mathbb{A}} + (\mathbb{L} \mathbb{B} z) \underline{\mathbb{A}} \right) + \sum_i^M {}_{M \setminus i} \left(\frac{{}_i \mathbb{A} \mathbb{B} \underline{\mathbb{A}}}{\underline{\mathbb{B}}} z \underline{\mathbb{A}} + \frac{{}_i \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} \mathbb{L}^z \underline{\mathbb{A}} \right) {}_M \mathbb{A}^z \left(\mathbb{B} \bowtie (\mathbb{B} \bowtie \mathbb{A}) \right)$$

$$= {}_M \mathbb{A} \mathbb{B} z \underline{\mathbb{B} \bowtie \mathbb{A}} + \sum_i^M {}_{M \setminus i} \frac{{}_i \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} z \left(\mathbb{B} \bowtie \mathbb{A} \right) = {}_M \mathbb{A} \mathbb{B} \left((\mathbb{B} z \underline{\mathbb{B}}) z \underline{\mathbb{A}} + (\mathbb{B} z \underline{\mathbb{B}} z) z \underline{\mathbb{A}} \right)$$

$$+ \sum_i^M {}_{M \setminus i} \left(\frac{({}_i \mathbb{A} \mathbb{B} z) \underline{\mathbb{B}} z}{\underline{\mathbb{B}}} z \underline{\mathbb{A}} + \frac{{}_i \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} \mathbb{B} z \underline{\mathbb{A}} + \frac{{}_i \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} \underline{\mathbb{B}} z \underline{\mathbb{A}} \right) + \sum_i^M {}_{M \setminus i} \left(\frac{{}_i \mathbb{A} \mathbb{B} \underline{\mathbb{B}} z}{\underline{\mathbb{B}}} z \underline{\mathbb{A}} - \sum_j^{M \setminus i} {}_{M \setminus i \setminus j} \frac{{}_j \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} z \underline{\mathbb{A}} \right)$$

$$= {}_M \mathbb{A} \mathbb{B} \left(\mathbb{B} z \underline{\mathbb{B}} z \right) z \underline{\mathbb{A}} + \sum_i^M {}_{M \setminus i} \frac{{}_i \mathbb{A} \mathbb{B} z \underline{\mathbb{B}} z + ({}_i \mathbb{A} \mathbb{B} z) \underline{\mathbb{B}} z}{\underline{\mathbb{B}}} z \underline{\mathbb{A}} = {}_M \mathbb{A} \mathbb{B} \left(\mathbb{B} \times \mathbb{B}' \right)_z z \underline{\mathbb{A}} + \sum_i^M {}_{M \setminus i} \frac{{}_i \mathbb{A} \mathbb{B} \times \mathbb{B}'}{\underline{\mathbb{B}}} z \underline{\mathbb{A}} = {}_M \mathbb{A}^z \left((\mathbb{B} \times \mathbb{B}') \bowtie \mathbb{A} \right)$$

$$\mathbb{B} \bowtie (\mathbb{A} \boxtimes \mathbb{A}') = (\mathbb{B} \bowtie \mathbb{A}) \boxtimes \mathbb{A}' + \mathbb{A} \boxtimes (\mathbb{B} \bowtie \mathbb{A}')$$

$${}_M \mathbb{A} \mathbb{B} \mathbb{B}_z \underline{\mathbb{A} \boxtimes \mathbb{A}'} + \sum_i^M {}_{M \setminus i} \frac{{}_i \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} \left(\mathbb{A} \boxtimes \mathbb{A}' \right) = \sum_{P \subset M} {}_{M \setminus P}^P \left(\left({}_P \mathbb{A} \mathbb{B} z \underline{\mathbb{A}} \right) \left({}_{M \setminus P} \mathbb{A}^z \underline{\mathbb{A}'} \right) + \left({}_P \mathbb{A}^z \underline{\mathbb{A}} \right) \left({}_{M \setminus P} \mathbb{A} \mathbb{B} z \underline{\mathbb{A}} \right) \right)$$

$$+ \sum_i^M {}_{M \setminus i} \left(\sum_{Q \subset M \setminus i} {}_{O: M \setminus Q \cup i}^{\bullet: Q} \frac{{}_i \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} z \underline{\mathbb{A}} \left({}_{M \setminus Q \cup i} \mathbb{A}^z \underline{\mathbb{A}'} \right) + {}_{\bullet: M \setminus Q \cup i}^{O: Q} \left({}_Q \mathbb{A}^z \underline{\mathbb{A}} \right) \frac{{}_i \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} z \underline{\mathbb{A}'} \right)$$

$$= (*) + \sum_{P \subset M} {}_{M \setminus P}^P \left(\sum_i^P {}_{P \setminus i} \frac{{}_i \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} z \underline{\mathbb{A}} \left({}_{M \setminus P} \mathbb{A}^z \underline{\mathbb{A}'} \right) + \sum_i^{M \setminus P} {}_{M \setminus P \cup i} \left({}_P \mathbb{A}^z \underline{\mathbb{A}} \right) \frac{{}_i \mathbb{A} \mathbb{B} z}{\underline{\mathbb{B}}} z \underline{\mathbb{A}'} \right)$$

$$= \sum_{P \subset M} {}_{M \setminus P}^P \left(\left({}_P \mathbb{A}^z (\mathbb{B} \bowtie \mathbb{A}) \right) \left({}_{M \setminus P} \mathbb{A}^z \underline{\mathbb{A}'} \right) + \left({}_P \mathbb{A}^z \underline{\mathbb{A}} \right) {}_{M \setminus P} \mathbb{A}^z \left(\mathbb{B} \bowtie \mathbb{A}' \right) \right) = {}_M \mathbb{A}^z \text{RHS}$$

$$(I) Q = P \setminus i \subset M \setminus i \Rightarrow {}_{M \setminus P \cup i} {}_{P \setminus i}^P = {}_{M \setminus Q \cup i} {}_Q^i = {}_{M \setminus Q \cup i} {}_{M \setminus Q \cup i} {}_Q^i = {}_{\emptyset: M \setminus Q \cup i} {}_{M \setminus i}^i$$

$$(\| \bowtie \|) \times (\|' \models \|) = (\| \times \|') \models$$

$${}_M\mathbf{\Delta}^z(\| \models \|) = \frac{\|_z}{M\mathbf{\Delta}} {}^z\| \Rightarrow {}_M\mathbf{\Delta}^z \underline{\|' \models \|} = \frac{\mathbf{L}\|_z}{M\mathbf{\Delta}} {}^z\| + \frac{\|_z}{M\mathbf{\Delta}} \underline{\|' \models \|} \Rightarrow$$

$${}_M\mathbf{\Delta}^z \left(\underline{\|} \left(d(\|' \models \|) \right) - \|' \models (\| \bowtie \|) \right) = {}_M\mathbf{\Delta}^z \underline{\|' \models \|} + \sum_i^M \mathcal{M}_{\leq i}^i \frac{i\mathbf{\Delta}\|_z}{M\mathbf{\Delta}} {}^z\left(\|' \models \| \right) - \frac{\|'_z}{M\mathbf{\Delta}} {}^z(\| \bowtie \|)$$

$$= \frac{\|_z \|'_z}{M\mathbf{\Delta}} {}^z\| + \frac{\|'_z}{M\mathbf{\Delta}} \underline{\|}_z {}^z\| + \sum_i^M \mathcal{M}_{\leq i}^i \left[\frac{\|'_z}{i\mathbf{\Delta}\|_z} \right] {}^z\| - \frac{\|'_z}{M\mathbf{\Delta}} \underline{\|}_z {}^z\| - \frac{\|'_z \|_z}{M\mathbf{\Delta}} {}^z\| - \sum_i^M \mathcal{M}_{\leq i}^i \left[\frac{i\mathbf{\Delta}\|_z}{\|'_z} \right] {}^z\| = \frac{\|_z \|'_z - \|'_z \|_z}{M\mathbf{\Delta}} {}^z\|$$