

$$\mathbb{R} \begin{array}{c} \nearrow \\ 0 \end{array} \overbrace{\mathbb{R}}^{\mathbb{R}} \begin{array}{c} \searrow \\ 0 \end{array} \mathbb{R} \longrightarrow \overline{\mathbb{R}} \begin{array}{c} \searrow \\ + \\ - \\ 0 \end{array} \mathbb{R}$$

$$\mathbb{R} \xrightarrow[\text{pos lin}]{\downarrow} \mathfrak{H}_{\Delta_0^0} \mathbb{R} \Rightarrow \downarrow_O := \bigcap_{0 \leq \gamma \leq 1_O} \downarrow \gamma \Rightarrow \bar{\mathbb{R}}_+ \xleftarrow[\text{top semipre-meas}]{\downarrow} \mathfrak{H}_{\Delta_0^0} \mathbb{2}$$

$\downarrow$  isoton

$$O \in \mathfrak{H} \Rightarrow \begin{cases} \bigvee g \in \mathfrak{H}_{\Delta_0^0} | 0|1 \\ 1_{\bar{O}} = g \end{cases} \quad 0 \leq \gamma \leq 1_O \Rightarrow \gamma \leq g \xrightarrow{\downarrow_{\text{pos}}} \downarrow \gamma \leq \downarrow g \Rightarrow \downarrow_O \leq \downarrow g < \infty$$

$$\begin{cases} \hat{O} \text{ disj} \\ 0 \leq \gamma \leq 1_{\hat{O}} \end{cases} \Rightarrow 0 \leq \gamma + \acute{\gamma} \leq 1_{\hat{O}} + 1_{\acute{O}} = 1_{O \cup \acute{O}} \Rightarrow \downarrow \gamma + \downarrow \acute{\gamma} = \downarrow \underbrace{\gamma + \acute{\gamma}} \leq \downarrow_{O \cup \acute{O}} \Rightarrow \downarrow_O + \downarrow_{\acute{O}} \leq \downarrow_{O \cup \acute{O}}$$

$$0 \leq \gamma \leq 1_{\bigcup_i^N O_i} \Rightarrow \bigvee_{\text{fin } N \subset \mathbb{N}} \gamma \leq 1_{\bigcup_i^N O_i} \xrightarrow{\text{PRO}} \bigwedge_j \bigvee 0 \leq \varphi_j \leq 1_{O_j} = \sum_j^N \varphi_j \mid \text{Trg } \gamma$$

$$\Rightarrow \gamma = \sum_j^N \gamma \varphi_j \Rightarrow \downarrow \gamma = \sum_j^N \downarrow \widehat{\gamma \varphi_j} \leq \sum_j^N \downarrow_{O_j} \leq \sum_i^N \downarrow_{O_i} \geq \downarrow_{\bigcup_i^N O_i}$$

$$0 \leq \gamma \leq 1_O \Rightarrow \bigvee_{V \in O} \text{Trg } \gamma \subset V \Rightarrow \downarrow \gamma \leq \downarrow_V \Rightarrow \downarrow_O \leq \bigvee_{V \in O} \downarrow_V$$

$$0 \leq \gamma \leq 1: \bigvee_{O \in \mathfrak{H}} \text{Trg } \gamma \subset O \quad \bigwedge_{0 \leq k \leq n+1} O_k = \begin{cases} \mathfrak{h} \in O \\ \mathfrak{h} \uparrow n > k-1 \end{cases} \Rightarrow \emptyset = O_{n+1} \in O_k \in O_0 = O$$

$$\gamma_k = (n\gamma - k) 1_{O_k \perp O_{k+1}} + 1_{O_k} = \begin{cases} 1 & \hat{O}_{k+1} \\ n\gamma - k + 1 & \hat{O}_k \perp O_{k+1} \\ 0 & \mathfrak{h} \perp O_k \end{cases} \in \mathfrak{H}_{\Delta_0^0} | 0|1 \leftarrow n\gamma = j \text{ on } \partial O_{j+1}$$

$$\gamma_k \mid O_{k+1} = 1 \Rightarrow \bigwedge 0 \leq \acute{\gamma} \leq O_{k+1} \Rightarrow \downarrow_{O_{k+1}} \leq \downarrow \gamma_k \leq \downarrow_{O_{k-1}} \leftarrow \text{Trg } \gamma_k \subset \hat{O}_k \subset O_{k-1}$$

$$1_{O_{k+1}} \leq \gamma_k \Rightarrow \downarrow_{O_{k+1}} \leq \int_{\downarrow}^{\mathfrak{h}} \gamma_k \leq \downarrow_{O_k} \leftarrow \gamma_k \leq 1_{O_k} \Rightarrow -\downarrow_{O_k} \leq -\int_{\downarrow}^{\mathfrak{h}} \gamma_k \leq -\downarrow_{O_{k+1}} \quad 1 \leq k \leq n \xrightarrow{\text{add}}$$

$$-\downarrow_{O_1} \leq \sum_{1 \leq k \leq n} \overbrace{\downarrow \gamma_k - \int_{\downarrow}^{\mathfrak{h}} \gamma_k} \leq \downarrow_{O_0} + \downarrow_{O_1} \sum_{1 \leq k \leq n} \gamma_k = n\gamma \Rightarrow -\frac{1}{n} \downarrow_{O_1} \leq \downarrow \gamma - \int_{\downarrow}^{\mathfrak{h}} \gamma \leq \frac{\downarrow_{O_0} + \downarrow_{O_1}}{n} \xrightarrow{n \rightarrow \infty} \downarrow \gamma = \int_{\downarrow}^{\mathfrak{h}} \gamma$$

$$\gamma = \gamma_+ - \gamma_- \Rightarrow \downarrow \gamma = \overline{\overline{\gamma_+}} \downarrow \frac{\gamma_+}{\overline{\overline{\gamma_+}}} - \overline{\overline{\gamma_-}} \downarrow \frac{\gamma_-}{\overline{\overline{\gamma_-}}} = \overline{\overline{\gamma_+}} \int_{\downarrow}^{\mathfrak{h}} \frac{\gamma_+}{\overline{\overline{\gamma_+}}} - \overline{\overline{\gamma_-}} \int_{\downarrow}^{\mathfrak{h}} \frac{\gamma_-}{\overline{\overline{\gamma_-}}} = \int_{\downarrow}^{\mathfrak{h}} \gamma_+ - \int_{\downarrow}^{\mathfrak{h}} \gamma_- = \int_{\downarrow}^{\mathfrak{h}} \gamma$$