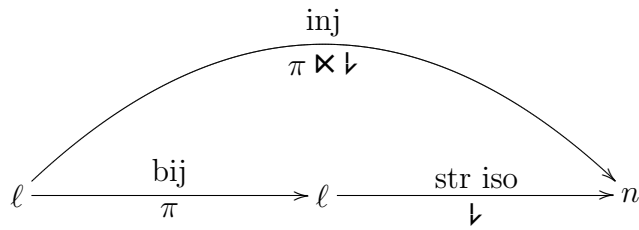


$${}^{\ell}\blacktriangleleft n = \{ \ell \xrightarrow[\text{inj}]{\mathfrak{l}} n \}$$

$${}^{\ell}\blacktriangleleft n$$

$$\Downarrow$$

$$\mathbf{C}(\ell) \times {}^{\ell}\blacktriangleleft n$$



$$\ell \xrightarrow[\text{inj}]{\mathfrak{l}} n \Rightarrow \ell \text{ set } {}^{\ell}\mathfrak{l} = \{ {}^0\mathfrak{l} \dots {}^{\ell-1}\mathfrak{l} \} \subset n$$

$$\Rightarrow \bigvee_{\text{eind}} \pi \in \mathbf{C}(\ell) \quad {}^{\pi^{-1}0}\mathfrak{l} < {}^{\pi^{-1}1}\mathfrak{l} < \dots < {}^{\pi^{-1}(\ell-1)}\mathfrak{l}$$

$$i_{\downarrow} = {}^{\pi^{-1}i}\mathfrak{l}$$

$$\# {}^{\ell}\blacktriangleleft n = {}_{\ell}(n) = n(n-1)\dots(n+1-\ell) \text{ falling power}$$

$$\text{LHS} = \underbrace{\mathbf{C}(\ell)}_{\#} \underbrace{{}^{\ell}\blacktriangleleft n}_{\#} = \ell! \begin{bmatrix} n \\ \ell \end{bmatrix} = \text{RHS}$$