

$$\begin{aligned}
x \square_{\bullet}^{\mu} &= x \mathfrak{E}^{\mu} \square_{\bullet} - \underbrace{x \mathfrak{E}^{\nu} \square_{\sigma}^i + \square_{\sigma}^i}_{\nu} \square_{\ell}^{\sigma \mu} = x \mathfrak{E}^{\nu} \underbrace{\nu \delta^{\mu} \square_{\bullet} - \square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{\nu} + \square_{\sigma}^i \square_{\ell}^{\sigma \mu} \\
\square_{\bullet}^{\mu} &= \mathfrak{E}^{\mu} \square_{\bullet} - \underbrace{\mathfrak{E}^{\nu} \square_{\sigma}^i + \square_{\sigma}^i}_{\nu} \square_{\ell}^{\sigma \mu} = \mathfrak{E}^{\nu} \underbrace{\nu \delta^{\mu} \square_{\bullet} - \square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{\nu} + \square_{\sigma}^i \square_{\ell}^{\sigma \mu}
\end{aligned}$$

$$\underbrace{\square_{\bullet}^{\mu}}_{\mu} \text{ conserved current } 0$$

$$\begin{aligned}
\text{LHS} &= \underbrace{\mathfrak{E}^{\nu} \nu \delta^{\mu} \square_{\bullet} - \square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{\mu} + \square_{\sigma}^i \square_{\ell}^{\sigma \mu} \\
&= \underbrace{x_{\mu} \mathfrak{E}^{\nu} \nu \delta^{\mu} \square_{\bullet} - \square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{\mu} + x \mathfrak{E}^{\nu} \underbrace{\nu \delta^{\mu} \square_{\bullet}}_{\mu} \underbrace{\square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{*} + \underbrace{\square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{\mu} \underbrace{\square_{\ell}^{\sigma \mu}}_{**} + \underbrace{\square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{\mu} \underbrace{\square_{\ell}^{\sigma \mu}}_{***} \\
\stackrel{\text{harm}}{=} & \underbrace{x_{\mu} \mathfrak{E}^{\nu} \nu \delta^{\mu} \square_{\bullet} - \square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{\mu} + x \mathfrak{E}^{\nu} \underbrace{\nu \square_{\bullet}}_{*} + \underbrace{\square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{**} + \underbrace{\square_{\sigma}^i \square_{\ell}^{\sigma \mu} + \square_{\mu}^j \square_{\tau}^m \square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{***} \\
&= \underbrace{x_{\mu} \mathfrak{E}^{\mu} \square_{\bullet} + x \mathfrak{E}^{\nu} \nu \square_{\bullet} + \square_{\sigma}^i \square_{\ell}^{\sigma \mu} + \square_{\sigma}^i \square_{\ell}^{\sigma \mu} + \square_{\mu}^j \square_{\tau}^m \square_{\sigma}^i \square_{\ell}^{\sigma \mu} - x_{\mu} \mathfrak{E}^{\nu} \nu \square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{\mu} \\
&= \underbrace{x_{\mu} \mathfrak{E}^{\mu} \square_{x: \square_{\bullet} \square_{\bullet}} + x \mathfrak{E}^{\nu} \square_{x: \square_{\bullet} \square_{\bullet}} + \square_{\sigma}^i \square_{x: \square_{\bullet}} \square_{\ell}^{\sigma \mu}}_{\mu} \\
&+ \underbrace{\square_{\sigma}^i \square_{x: \square_{\bullet}} \square_{\ell}^{\sigma \mu} + \square_{\mu \tau}^j \square_{\sigma}^m \square_{x: \square_{\bullet}} \square_{\ell}^{\sigma \mu} - x_{\mu} \mathfrak{E}^{\nu} \nu \square_{\sigma}^i \square_{\ell}^{\sigma \mu}}_{\mu} = 0
\end{aligned}$$

$$\left\{ \begin{array}{c} x \\ \sigma \mathbb{F}_\ell^i \\ \mu \sigma \mathbb{F}_\ell^i \end{array} \right\} \times \underbrace{\left\{ \begin{array}{c} \mathbb{F} \\ \mathbb{F} \end{array} \right\}}_{\times} \times \underbrace{\left\{ \begin{array}{c} \mathbb{F} \\ \mathbb{F} \end{array} \right\}}_{\times} = \left\{ \begin{array}{c} x \\ \sigma \mathbb{F}_\ell^i \\ \mu \sigma \mathbb{F}_\ell^i \end{array} \right\} \times \underbrace{\left\{ \begin{array}{c} \mathbb{F} \\ \mathbb{F} \\ \mathbb{F} \\ \mathbb{F} \end{array} \right\}}_{\times}$$

$$\text{LHS} = \left\{ \begin{array}{c} x \mathbb{F} \\ x \mathbb{F}_\ell^i \\ x \mathbb{F}_\ell^{-1} \nu^x \end{array} \right\} \underbrace{\left\{ \begin{array}{c} \partial_{\nu \sigma \mathbb{F}_\ell^i} \\ \partial_j^{\tau} \end{array} \right\}}_{\times} + \underbrace{\left\{ \begin{array}{c} m \\ \nu \tau \mathbb{F}_m^j \end{array} \right\}}_{\times} \times \underbrace{\left\{ \begin{array}{c} \mathbb{F} \\ \mathbb{F} \end{array} \right\}}_{\times} = \left\{ \begin{array}{c} x \mathbb{F} \\ x \mathbb{F}_\ell^i x \mathbb{F}_\ell^i \\ x \mathbb{F}_\ell^{-1} \nu \end{array} \right\} \left(\underbrace{\left\{ \begin{array}{c} \partial_{\nu \mathbb{F}_\ell^i} \\ \partial_j^{\tau} \end{array} \right\}}_{\times} + \underbrace{\left\{ \begin{array}{c} m \\ \nu \mathbb{F}_\ell^i \end{array} \right\}}_{\times} \partial_j^{\tau} \underbrace{\left\{ \begin{array}{c} x \mathbb{F}_\ell^i \\ x \mathbb{F}_\ell^{-1} \nu \end{array} \right\}}_{\times} \partial_{\lambda \tau \mathbb{F}_m^j} + \underbrace{\left\{ \begin{array}{c} j \\ \tau \mathbb{F}_m^j \end{array} \right\}}_{\times} \partial_{k \mathbb{F}_\ell^i} \right)$$

$$\text{RHS} = \left\{ \begin{array}{c} x \mathbb{F} \\ x \mathbb{F}_\ell^i x \mathbb{F}_\ell^i \\ x \mathbb{F}_\ell^{-1} \nu \lambda^x \end{array} \right\} \underbrace{\left\{ \begin{array}{c} \partial_{\lambda \mathbb{F}_\ell^i} \\ \partial_{k \mathbb{F}_\ell^i} \end{array} \right\}}_{\times} + \underbrace{\left\{ \begin{array}{c} m \\ \lambda \mathbb{F}_\ell^i \end{array} \right\}}_{\times} \partial_{k \mathbb{F}_\ell^i} \underbrace{\left\{ \begin{array}{c} \mathbb{F} \\ \mathbb{F} \end{array} \right\}}_{\times}$$

$$= \left\{ \begin{array}{c} x \mathbb{F} \\ x \mathbb{F}_\ell^i x \mathbb{F}_\ell^i \\ x \mathbb{F}_\ell^{-1} \nu x \mathbb{F}_\ell^{-1} \lambda \end{array} \right\} \left(\underbrace{\left\{ \begin{array}{c} x \mathbb{F} \\ \lambda \mathbb{F}_\ell^i \end{array} \right\}}_{\times} \partial_{\mathbb{F}_\ell^i} + \underbrace{\left\{ \begin{array}{c} m \\ \mathbb{F}_\ell^i \end{array} \right\}}_{\times} \partial_j^{\tau} \underbrace{\left\{ \begin{array}{c} j \\ \lambda \tau \mathbb{F}_m^j \end{array} \right\}}_{\times} + \underbrace{\left\{ \begin{array}{c} m \\ \mathbb{F}_\ell^i \end{array} \right\}}_{\times} \partial_j^{\tau} \underbrace{\left\{ \begin{array}{c} j \\ \tau \mathbb{F}_m^j \end{array} \right\}}_{\times} \partial_{k \mathbb{F}_\ell^i} \right)$$