$$\boxed{x:\Upsilon:\Psi} = \boxed{x^\mu : {}_\sigma\Upsilon^i{}_\ell : {}_{\mu\sigma}\Psi^i{}_\ell} \overset{\text{group}}{\underset{\text{inv}}{=}} \det {}^x\mathfrak{s} \boxed{{}^x\mathfrak{s} : {}_\sigma\boxed{x:\Upsilon}^i{}_\ell : {}^{x}{}_{\mu}\mathfrak{s}^{-1\nu} \overbrace{{}_\sigma\boxed{x:\Upsilon}^i_\nu{}_\ell + {}_{\nu\tau}\Psi^j{}_m {}^m{}_\sigma\boxed{x:\Upsilon}^i_j{}^\tau{}_\ell}}$$

$$0 \overset{\text{Lie alg}}{\underset{\text{inv}}{=}} {}^x{}_\mu\mathfrak{g}^\mu \boxed{x:\Upsilon:\Psi} + {}^x\mathfrak{g}^\nu {}_\nu\boxed{x:\Upsilon:\Psi} + {}_\sigma\boxed{x:\Upsilon}^i{}_\ell^\bullet {}^\ell\boxed{x:\Upsilon:\Psi}_i^\sigma + {}_\sigma\boxed{x:\Upsilon}^i_\mu{}_\ell^\bullet + {}_{\mu\tau}\Psi^j{}_m {}^m{}_\sigma\boxed{x:\Upsilon}^i_j{}^\tau{}_\ell^\bullet - {}^x{}_\mu\mathfrak{g}^\nu {}_{\nu\sigma}\Psi^i{}_\ell {}^\ell\boxed{x:\Upsilon:\Psi}_i^{\sigma\mu}$$

$${}_\sigma\boxed{x:\Upsilon}^i{}_\ell^\bullet = \frac{d}{dt} {}_\sigma\boxed{x:\Upsilon}^i t_\ell$$

$${}^x\mathfrak{g} = \frac{d}{dt} {}^x\mathfrak{g} \Longrightarrow \frac{d}{dt} \det {}^x\mathfrak{g} = \operatorname{tr} \frac{d}{dt} {}^x\mathfrak{g} = \operatorname{tr} {}^x\mathfrak{g} = {}^x{}_\mu\mathfrak{g}^\mu$$

$$0 = \frac{d}{dt}\boxed{x:\Upsilon:\Psi} = \frac{d}{dt}\det {}^x\mathfrak{g} \boxed{{}^x\mathfrak{g} : {}_\sigma\boxed{x:\Upsilon}^i t_\ell : {}^x{}_\mu\mathfrak{g}^{-1\nu} \underbrace{{}_\sigma\boxed{x:\Upsilon}^i_\nu t_\ell + {}_{\nu\tau}\Psi^j{}_m {}^m{}_\sigma\boxed{x:\Upsilon}^i_j{}^\tau t_\ell}}$$

$$= \frac{d}{dt}\det {}^x\mathfrak{g} \boxed{x:\Upsilon:\Psi} + 1 \cdot \frac{d}{dt}\boxed{{}^x\mathfrak{g} : {}_\sigma\boxed{x:\Upsilon}^i t_\ell : {}^x{}_\mu\mathfrak{g}^{-1\nu} \underbrace{{}_\sigma\boxed{x:\Upsilon}^i_\nu t_\ell + {}_{\nu\tau}\Psi^j{}_m {}^m{}_\sigma\boxed{x:\Upsilon}^i_j{}^\tau t_\ell}}$$

$$= {}^x{}_\mu\mathfrak{g}^\mu \boxed{x:\Upsilon:\Psi} + {}^x\mathfrak{g}^\nu {}_\nu\boxed{x:\Upsilon:\Psi} + {}_\sigma\boxed{x:\Upsilon}^i{}_\ell^\bullet {}^\ell\boxed{x:\Upsilon:\Psi}_i^\sigma + \underbrace{{}_\sigma\boxed{x:\Upsilon}^i_\mu{}_\ell^\bullet + {}_{\mu\tau}\Psi^j{}_m {}^m{}_\sigma\boxed{x:\Upsilon}^i_j{}^\tau{}_\ell^\bullet - {}^x{}_\mu\mathfrak{g}^\nu {}_{\nu\sigma}\Psi^i{}_\ell} {}^\ell\boxed{x:\Upsilon:\Psi}_i^{\sigma\mu} = \text{RHS}$$

$$x^\nu : {}_\sigma\Upsilon^i{}_\ell : {}_{\mu\sigma}\Psi^i{}_\ell \in \mathbb{R}^d \times {}_d\mathbb{R}^N_M \times {}_{d^2}\mathbb{R}^N_M \xrightarrow[\text{current}]{\mathcal{J}^\mu} \mathbb{R} \ni \boxed{x : {}_\sigma\Upsilon^i{}_\ell : {}_{\mu\sigma}\Psi^i{}_\ell}^{\mu\,\bullet}$$

$$\boxed{x:\Upsilon:\Psi}^{\mu\,\bullet} = {}^x\mathfrak{g}^\mu \boxed{x:\Upsilon:\Psi} - \underbrace{{}^x\mathfrak{g}^\nu {}_{\nu\sigma}\Psi^i{}_\ell + {}_\sigma\boxed{x:\Upsilon}^i{}_\ell^\bullet} {}^\ell\boxed{x:\Upsilon:\Psi}_i^{\sigma\mu} = {}^x\mathfrak{g}^\nu {}_\nu\delta^\mu \boxed{x:\Upsilon:\Psi} - {}_{\nu\sigma}\Psi^i{}_\ell {}^\ell\boxed{x:\Upsilon:\Psi}_i^{\sigma\mu} + {}_\sigma\boxed{x:\Upsilon}^i{}_\ell^\bullet {}^\ell\boxed{x:\Upsilon:\Psi}_i^{\sigma\mu}$$