

$$\begin{aligned} \bar{\mathfrak{h}}_{\triangleleft 2} &= \{U \subset \bar{\mathfrak{h}}\} \xrightarrow[\text{compl}]{\bar{()}} \bar{\mathfrak{h}}_{\triangleleft 2} \\ \text{rel compl} &\begin{cases} V \perp U & = \underbrace{x \in V \wedge x \notin U}_x \\ x \in V \perp U & \Leftrightarrow x \in V \wedge x \notin U \end{cases} \\ \text{abs compl} &\begin{cases} \bar{V} & = \underbrace{x \notin V}_x \\ x \in \bar{V} & \Leftrightarrow x \notin V \end{cases} \end{aligned}$$

$$\text{de Morgan / } \quad \overline{U \cap V} = \bar{U} \cup \bar{V}$$

$$\begin{aligned} \subset : & x \in \overline{U \cap V} \Rightarrow x \notin U \cap V \\ \vee \begin{cases} \text{if } x \in \bar{U} & \Rightarrow x \in \bar{U} \cup \bar{V} \\ \text{if } x \notin \bar{U} & \Rightarrow x \in U \Rightarrow x \notin V \Rightarrow x \in \bar{V} \Rightarrow x \in \bar{U} \cup \bar{V} \end{cases} & \Rightarrow x \in \bar{U} \cup \bar{V} \\ \supset : & x \in \bar{U} \cup \bar{V} \Rightarrow \vee \begin{cases} x \in \bar{U} & x \notin U \Rightarrow x \notin U \cap V \Rightarrow x \in \overline{U \cap V} \\ x \in \bar{V} & x \notin V \Rightarrow x \notin U \cap V \Rightarrow x \in \overline{U \cap V} \end{cases} \end{aligned}$$

$$\text{de Morgan / } \quad \overline{\bar{U} \cup \bar{V}} = \bar{U} \cap \bar{V}$$

$$\begin{aligned} \subset : & x \in \overline{\bar{U} \cup \bar{V}} \Rightarrow x \notin \bar{U} \cup \bar{V} \Rightarrow \wedge \begin{cases} x \notin U & \Leftarrow U \subset \bar{U} \cup \bar{V} \\ x \notin V & \Leftarrow V \subset \bar{U} \cup \bar{V} \end{cases} \Rightarrow x \in \bar{U} \cap \bar{V} \\ \supset : & x \in \bar{U} \cap \bar{V} \Rightarrow \wedge \begin{cases} x \in \bar{U} & \Rightarrow x \notin U \\ x \in \bar{V} & \Rightarrow x \notin V \end{cases} \\ \nexists x \in \bar{U} \cup \bar{V} & \Rightarrow \vee \begin{cases} x \in U & \nexists \\ x \in V & \nexists \end{cases} \Rightarrow x \notin \bar{U} \cup \bar{V} \Rightarrow x \in \overline{\bar{U} \cup \bar{V}} \end{aligned}$$

family de Morgan /  $\overline{\bigcap_i U_i} = \bigcup_i \bar{U}_i$

family de Morgan /  $\overline{\bigcup_i U_i} = \bigcap_i \bar{U}_i$

$$\subset : x \in \overline{\bigcup_i U_i} \Rightarrow x \notin \bigcup_i U_i \Rightarrow \overline{\bigvee_i x \in U_i} \Rightarrow \begin{cases} x \notin U_i \\ \bigwedge_i \end{cases} \Rightarrow x \in \bar{U}_i \Rightarrow x \in \bigcap_i \bar{U}_i$$

$$\supset : x \in \bigcap_i \bar{U}_i \Rightarrow \begin{cases} x \in \bar{U}_i \\ \bigwedge_i \end{cases} \Rightarrow x \notin U_i$$

$$\nexists x \notin \overline{\bigcup_i U_i} \Rightarrow x \in \bigcup_i U_i \Rightarrow \bigvee_{i_0} x \in U_{i_0} \nexists$$