

$$\mathbb{h}^{\infty} \begin{array}{c} \nearrow \\ \hline \searrow \end{array} \begin{array}{c} \mathbb{h} \\ \hline \mathbb{J} \end{array} \xleftarrow{\mathbb{l}'} \underbrace{\mathbb{h}^{\infty} \begin{array}{c} \nearrow \\ \hline \searrow \end{array} \mathbb{J}}_{2^n}$$

$$\mathbb{h}^{\infty} \begin{array}{c} \nearrow \\ \hline \searrow \end{array} \begin{array}{c} \mathbb{h} \\ \hline \mathbb{J} \end{array} \ni \mathbb{l}^J = \sum_{j \in J} \mathbb{l}^j \quad \text{dual standard basis}$$

$$\mathbb{l}^I \mathbb{l}^J = \det \mathbb{l}^i \mathbb{l}^j = \det {}_i \delta^j = {}_I \delta^J = {}_I \mathbb{l} \mathbb{l}^J$$

$$\mathbb{l}^I = {}_I \mathbb{l}$$

$$\mathbb{l}^I \times \mathbb{l}^J = \mathbb{l}^I \mathring{\eta} \mathbb{l}^J = \det \mathbb{l}^i \times \mathbb{l}^j = \det {}_i \eta^j = {}_I \eta^J = {}_I \mathring{\eta}^J$$

$$\times {}_I \mathbb{l} = \mathbb{l}^I {}_I \eta^I$$

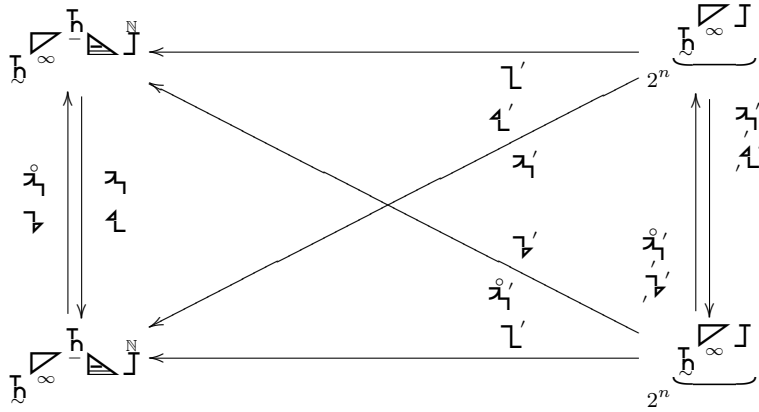
$$\mathbb{l}^I = (\times {}_I \mathbb{l}) {}_I \eta^I$$

$$\ast \mathbb{l}^I = \mathbb{l}^{N-I} \overline{{}_I \delta^{N-I}} {}_I \eta^I$$

$${}_{\mu} \mathbb{l} = {}_I \mathbb{l} \underbrace{\mathbb{l}^I}_{\mu} \mathbb{l}^N = \det ({}_{\mu} \mathbb{l} \mathbb{l}^{\nu}) = \det {}_{\mu} \delta^{\nu} = {}_M \delta^N$$

$$\mathbb{h}^I = {}_I \mathbb{l} \underbrace{\mathbb{l}^I}_{\mathbb{h}^I} \mathbb{l}^J = \det \mathbb{l}^i \mathbb{l}^j = \det {}_i \delta^j = {}_I \delta^J = {}_I \mathbb{l} \mathbb{l}^J$$

$$\mathbb{l}^I = {}_I \mathbb{l}$$



$$\mathbb{l}^I \times_{\mathbb{h}} \mathbb{l}^J = \begin{cases} \mathbb{l}^I \mathbb{h} \mathbb{l}^J = \mathbb{h}^{IJ} \\ \mathbb{l}^I \mathbb{l}^J \mathbb{l}^J = {}_I \mathbb{l} \mathbb{l}^J \mathbb{l}^J = \det \mathbb{l}^i \times_{\mathbb{h}} \mathbb{l}^j = \det {}_I \mathbb{l}^J = {}_I \mathbb{l}^J \end{cases}$$

$$\mathbb{x}^J \mathbb{l} = \sum_{|I|=|J|} \mathbb{l}^I \mathbb{x}'_I \mathbb{l}^J$$

$$\mathbb{1}^J = \sum_{|I|=|J|} \left( \overset{\circ}{\mathbb{X}}_I \mathbb{1} \right) {}_I \mathbb{1}_z^J$$

$${}_M \mathbb{1} \sum_I \mathbb{1}^I {}_I \overset{\circ}{\mathbb{A}}^J = \sum_I {}_M \delta^I {}_I \overset{\circ}{\mathbb{A}}^J = {}_M \overset{\circ}{\mathbb{A}}^J = {}_M \mathbb{1} \overset{\circ}{\mathbb{X}}_J \mathbb{1}$$

$${}_M \mathbb{1} \mathbb{1}^J = {}_M \delta^J = \det_M \left( \mathbb{1}_z^z \mathbb{A} \right)^J = \sum_{|I|=|J|} {}_M \mathbb{1}_z^I {}_I \mathbb{A}^J = \sum_I ({}_M \mathbb{1} \times \overset{\circ}{\mathbb{X}}_I \mathbb{1}) {}_I \mathbb{A}^J$$

$$\overset{\circ}{\mathbb{X}}_I \mathbb{1} = \mathbb{A}^I {}_I \eta^I$$

$$\mathbb{X} \mathbb{A}^I = ({}_I \mathbb{1}) {}_I \eta^I$$

$$\mathbb{H} \overset{\circ}{\mathbb{A}}^J \ni \begin{cases} \mathbb{A}^J & = \mathbb{A}_9 \mathbb{1} \\ \mathbb{A}^J & = \sum_{j \in J} \overset{\circ}{\mathbb{X}}_j \mathbb{A}^j = \mathbb{A} \mathbb{1}^J \end{cases} \text{ dual ONBasis}$$

$$\begin{cases} \mathbb{A}_1 & = \mathbb{A}_1^I \mathbb{1} \\ \mathbb{A} & = \mathbb{A}^I {}_I \mathbb{1} \end{cases}$$

$$\begin{cases} \mathbb{A}_1^I \mathbb{X}_h \mathbb{A}^J = \mathbb{A}_1^I \mathbb{A} \mathbb{A}^J = \mathbb{A}_1^I \underbrace{\mathbb{A}_1^{\circ} \overset{\circ}{\mathbb{A}}^J}_{\mathbb{A}_1^{\circ} \mathbb{A}^J} = \mathbb{A}_1^I \mathbb{A}_1^{\circ} \overset{\circ}{\mathbb{A}}^J \mathbb{A}_1^{\circ} & = \mathbb{A}_1 \mathbb{1}^I \mathbb{A}_1^{\circ} \mathbb{A}_1^{\circ} \\ \mathbb{A}^I \mathbb{X}_h \mathbb{A}^J = \det \mathbb{A}^i \left( \mathbb{1}^* \eta \mathbb{1} \right) \mathbb{A}^j = \det \left( \mathbb{1}^* \mathbb{A}^i \right) \eta \left( \mathbb{1}^* \mathbb{A}^j \right) = \det {}_i \mathbb{1} \eta \mathbb{1}^j = \mathbb{A}^I \mathbb{1} \mathbb{A}^J = \mathbb{A}^I \underbrace{\mathbb{1}^* \overset{\circ}{\mathbb{A}}^J}_{\mathbb{1}^* \mathbb{A}^J} \mathbb{A}^J = \mathbb{1} \mathbb{A}^I \overset{\circ}{\mathbb{A}}^J \mathbb{A}^J & = \mathbb{A} \mathbb{1}^I \mathbb{1}^* \overset{\circ}{\mathbb{A}}^J \end{cases}$$

$$\overset{*}{\mathbb{X}}_z \mathbb{A}^I = \mathbb{A}^{N-I} \overline{{}_I \mathbb{1}^* \mathbb{A}^I} \eta^I$$

$$\overset{*}{\mathbb{X}}_z \mathbb{1}^J = \sum_{|I|=|J|} \mathbb{1}^{N-I} \overline{{}_I \mathbb{1}^* \mathbb{A}^I} {}_I \mathbb{1}_z^J \left( {}_N \eta^N / {}_N \mathbb{1}_z^N \right)^{1/2}$$

$$\mathbb{A}^N = c \mathbb{1}^N$$

$${}_N \eta^N = \mathbb{A}^N \overset{*}{\mathbb{X}}_z \mathbb{A}^N = c^2 \mathbb{1}^N \overset{*}{\mathbb{X}}_z \mathbb{1}^N = c^2 {}_N \mathbb{1}_z^N \Rightarrow \text{LHS} = \sum_I \underbrace{\mathbb{1} \mathbb{1} \mathbb{A}^N}_{\mathbb{1} \mathbb{1} \mathbb{A}^N} {}_I \mathbb{1}_z^J = \sum_I \mathbb{1} \mathbb{1} \mathbb{A}^N \left( {}_N \eta^N / {}_N \mathbb{1}_z^N \right)^{1/2} {}_I \mathbb{1}_z^J = \text{RHS}$$

$$\mathbb{A}^I = \begin{cases} \mathbb{A}_1^{\circ} \mathbb{A}_1^I \mathbb{A}_1^{\circ} \\ \mathbb{A} \mathbb{1}^I \mathbb{A} \end{cases}$$

$${}_I \delta^J = \begin{cases} \mathbb{A}_1^{\circ} \mathbb{A}_1^J \\ \mathbb{A} \mathbb{1}^J \mathbb{A} \end{cases}$$

$$\mathbb{A} = \begin{cases} \mathbb{A}_1^{\circ} \mathbb{A}_1^{\circ} \mathbb{A} \\ \mathbb{A} \mathbb{1}^{\circ} \mathbb{A} \end{cases}$$

$${}_M \delta^N = \begin{cases} \mathfrak{z}_M \mathfrak{z}^N \\ \mathfrak{z}_M \mathfrak{z}^N \end{cases}$$

$$\mathfrak{z}' \mathfrak{z} = \begin{cases} \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \\ \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \end{cases}$$

$$\mathfrak{z}^J = \begin{cases} \mathfrak{z}^J \mathfrak{z}^J = \mathfrak{z}^L \mathfrak{z}^J \\ \mathfrak{z}^J \mathfrak{z}^J = \mathfrak{z}^L \mathfrak{z}^J \end{cases}$$

$$\mathfrak{z}' \mathfrak{z}' = \begin{cases} \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \\ \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \end{cases}$$

$$\mathfrak{z}^N = \begin{cases} \mathfrak{z}^N \mathfrak{z}^N = \mathfrak{z}^K \mathfrak{z}^N \\ \mathfrak{z}^N \mathfrak{z}^N = \mathfrak{z}^K \mathfrak{z}^N \end{cases}$$

$$\begin{cases} \mathfrak{z}' \mathfrak{z} = \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \\ \mathfrak{z}' \mathfrak{z} = \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \end{cases}$$

$$\begin{cases} \mathfrak{z}^J = \mathfrak{z}^L \mathfrak{z}^J = \mathfrak{z}^L \mathfrak{z}^J \\ \mathfrak{z}^J = \mathfrak{z}^L \mathfrak{z}^J = \mathfrak{z}^L \mathfrak{z}^J \end{cases}$$

$$\begin{cases} \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \\ \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \end{cases}$$

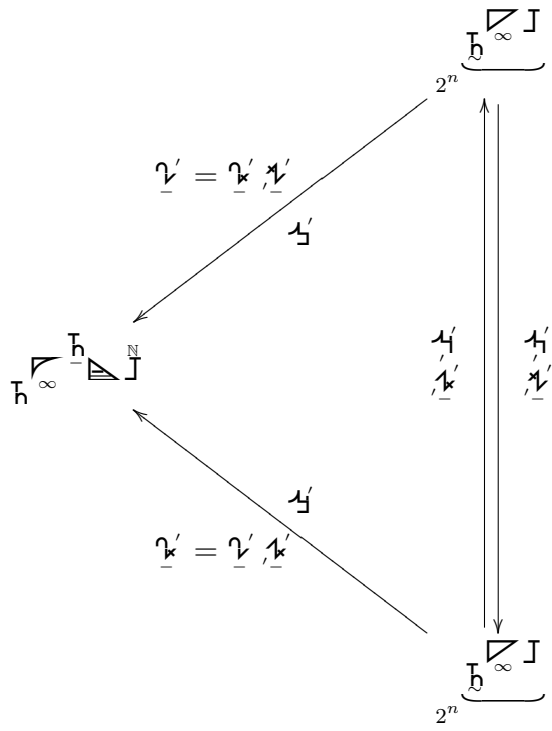
$$\begin{cases} \mathfrak{z}^N = \mathfrak{z}^K \mathfrak{z}^N = \mathfrak{z}^K \mathfrak{z}^N \\ \mathfrak{z}^N = \mathfrak{z}^K \mathfrak{z}^N = \mathfrak{z}^K \mathfrak{z}^N \end{cases}$$

$$\begin{cases} \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \\ \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \end{cases}$$

$$\begin{cases} \mathfrak{z}_M^J = \mathfrak{z}_M \mathfrak{z}^J = \mathfrak{z}_M \mathfrak{z}^J \\ \mathfrak{z}_M^J = \mathfrak{z}_M \mathfrak{z}^J = \mathfrak{z}_M \mathfrak{z}^J \end{cases}$$

$$\begin{cases} \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \\ \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' = \mathfrak{z}' \mathfrak{z}' \end{cases}$$

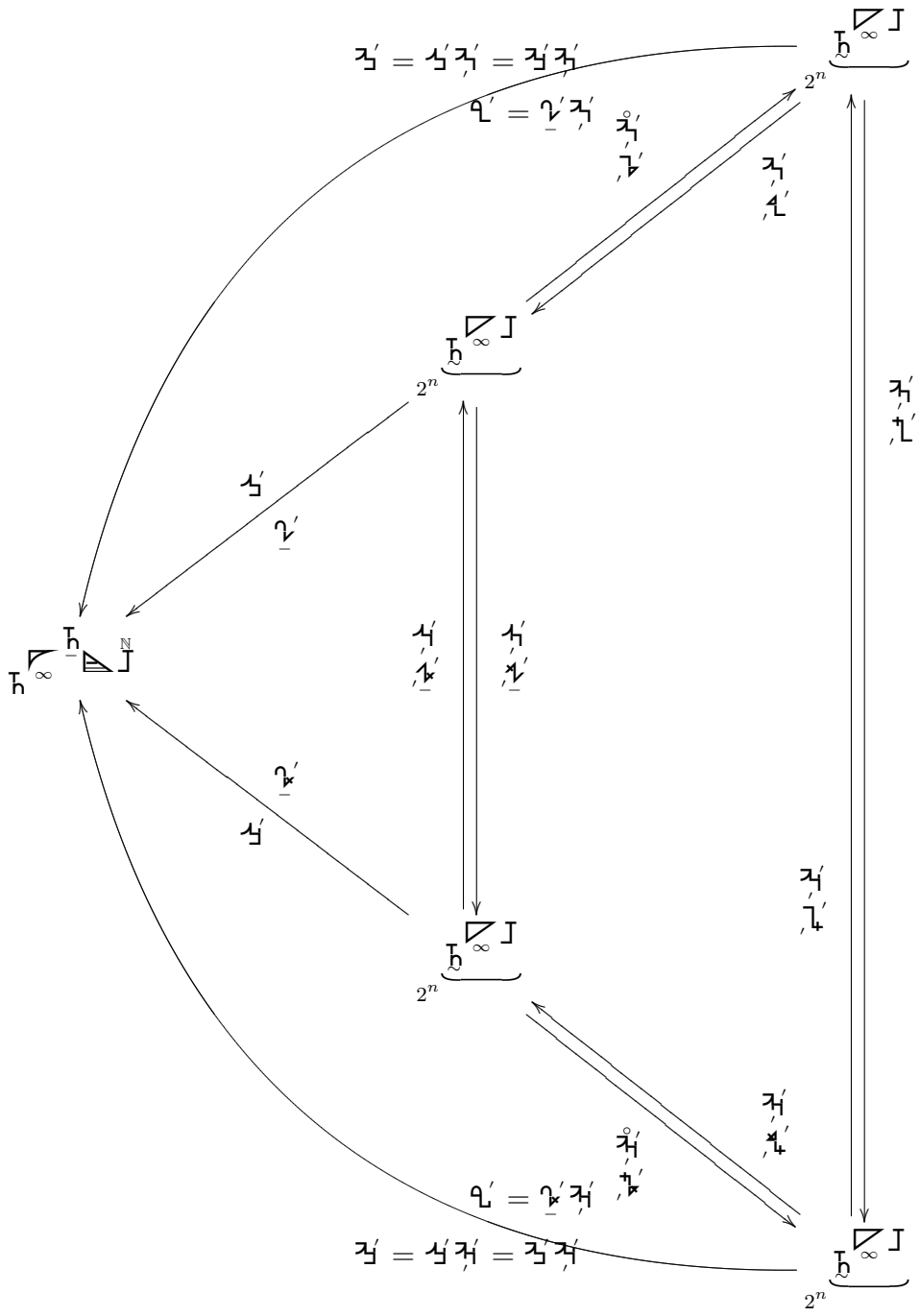
$$\begin{cases} \mathfrak{z}_I^N = \mathfrak{z}_I \mathfrak{z}^N = \mathfrak{z}_I \mathfrak{z}^N \\ \mathfrak{z}_I^N = \mathfrak{z}_I \mathfrak{z}^N = \mathfrak{z}_I \mathfrak{z}^N \end{cases}$$



$\begin{array}{|c|} \hline \square \\ \hline \end{array} \begin{array}{l} \nearrow \\ \text{J} \end{array} \leftarrow \underline{\underline{\nu}}^j$  holonomic basis

$$1 = \underline{\underline{\nu}}^1$$

$${}_M \delta^N = \underline{\underline{\nu}}^N$$



$$\mathcal{H}_\infty^N \cong \begin{cases} \mathcal{U}^J & * \\ \mathcal{U}^I & = \sum_{j \in J} \mathcal{U}^j \end{cases} \text{ dual ONbasis}$$

$$\mathcal{U}^I \mathcal{U}^J = \delta^{IJ}$$

$$\mathbf{x}_I \mathbf{b} = \mathbf{a}_I \eta^I$$

$$\mathbf{a}_I = (\mathbf{x}_I \mathbf{b}) \eta^I$$

$$* \mathbf{a}^I = \mathbf{a}^{N-I} \overline{I > \overline{N-I}} \eta^I$$

$$\mathbf{a}_I = \begin{cases} \mathbf{x}_I \mathbf{b}_I \\ \mathbf{b}_I \mathbf{a}_I \end{cases}$$

$${}_I \delta^J = \begin{cases} \mathbf{x}_I \mathbf{x}_J \\ \mathbf{b}_I \mathbf{b}_J \end{cases}$$

$$\begin{cases} \mathbf{x}_I \mathbf{a}_I = \mathbf{a}_I \mathbf{x}_I \\ \mathbf{a}_I \mathbf{a}_I = \mathbf{a}_I \mathbf{a}_I \end{cases}$$

$$\begin{cases} \mathbf{x}_I^J = \mathbf{a}_I^L \mathbf{x}_I^J \\ \mathbf{a}_I^J = \mathbf{a}_I^L \mathbf{a}_I^J \end{cases}$$

$$\mathbf{a}_I^J = \begin{cases} \mathbf{x}_I \mathbf{a}_I^J \\ \mathbf{a}_I \mathbf{b}_I^J \end{cases}$$

$$\mathbf{a}_I^N = \begin{cases} \mathbf{x}_I^K \mathbf{a}_I^N \\ \mathbf{a}_I^K \mathbf{b}_I^N \end{cases}$$

$$\begin{cases} \mathbf{x}_I \mathbf{a}_I = \mathbf{a}_I \mathbf{x}_I \\ \mathbf{a}_I \mathbf{a}_I = \mathbf{a}_I \mathbf{a}_I \end{cases}$$

$$\begin{cases} \mathbf{x}_I^J = \mathbf{a}_I \mathbf{x}_I^J \\ \mathbf{a}_I^J = \mathbf{a}_I \mathbf{a}_I^J \end{cases}$$

$$\begin{cases} \mathbf{a}_I^J = \mathbf{x}_I \mathbf{a}_I^J \\ \mathbf{b}_I^J = \mathbf{b}_I \mathbf{a}_I^J \end{cases}$$

$$\begin{cases} \mathbf{a}_I^N = \mathbf{x}_I \mathbf{a}_I^N \\ \mathbf{b}_I^N = \mathbf{b}_I \mathbf{a}_I^N \end{cases}$$

