

$$\begin{array}{ccc}
\mathbb{H} \begin{array}{c} \triangle \\ \circ \end{array} \mathbb{R}i \rtimes \mathbb{H} & \xrightarrow{\cong} & \mathbb{H} \rtimes \mathbb{R}i \begin{array}{c} \triangle \\ \mathbb{H} \end{array} \\
\downarrow M \times C & & \downarrow \hat{M} \times \hat{C} \\
\mathbb{U} | \begin{array}{c} \mathbb{H} \\ \triangle \\ m \end{array} \mathbb{C} & \xrightarrow{\cong} & \mathbb{U} | \begin{array}{c} \mathbb{H} \\ \triangle \\ m \end{array} \mathbb{C}
\end{array}$$

$$\begin{array}{ccc}
\begin{array}{c} \mathbb{H} \\ \triangle \\ m \end{array} \mathbb{C} & \xleftarrow[\alpha_{\delta \mathbb{X}}^{\infty}]{\ell_{\zeta}} & \begin{array}{c} \mathbb{H} \\ \triangle \\ m \end{array} \mathbb{C} \\
\downarrow \cong & & \downarrow \cong \\
\begin{array}{c} \mathbb{H} \\ \triangle \\ m \end{array} \mathbb{C} & \xleftarrow[\alpha_{\delta \mathbb{X}}^1]{M_{\bar{\zeta}}} & \begin{array}{c} \mathbb{H} \\ \triangle \\ m \end{array} \mathbb{C}
\end{array}$$

multiplications $\mathbb{H} \xrightarrow[\text{hom}]{M} \mathbb{U} | \begin{array}{c} \mathbb{H} \\ \triangle \\ m \end{array} \mathbb{C}$

$$\zeta \overbrace{n \rtimes \eta} = \zeta_n \zeta \eta$$

$$\ell_{s^t} \eta = \bar{s}^t \eta \text{ translations}$$