$$\begin{array}{c|c|c} a & x & \xi \\ \hline 0 & -a^* & 0 \\ \hline 0 & \xi^* & 0 \end{array}$$

$$\begin{array}{c|c|c} a & x & \xi \\ \hline 0 & -a^* & 0 \\ \hline 0 & \xi^* & 0 \end{array} \ast \begin{array}{c|c|c} b & y & \eta \\ \hline 0 & -b^* & 0 \\ \hline 0 & \eta^* & 0 \end{array} = \begin{array}{c|c|c} ab-ba & ay+ya^*-xb^*-bx+\xi\eta^*+\eta\xi^* & a\eta-b\xi \\ \hline 0 & a^*b^*-b^*a^* & 0 \\ \hline 0 & \eta^*a^*-\xi^*b^* & 0 \end{array}$$

$$\begin{array}{c|c|c} 0 & x & \xi \\ \hline 0 & 0 & 0 \\ \hline 0 & \xi^* & 0 \end{array} \ast \begin{array}{c|c|c} 0 & y & \eta \\ \hline 0 & 0 & 0 \\ \hline 0 & \eta^* & 0 \end{array} = \begin{array}{c|c|c} 0 & \xi\eta^*+\eta\xi^* & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}$$

$$\xi^*\,\sigma^\mu\,\eta = 2\overline{i\sigma^2\xi}\,\overbrace{i\sigma^2\eta}^{T}$$

$$x^0 = \xi^*\,\sigma^0\,\eta = \begin{bmatrix}\bar\xi_1 & \bar\xi_2\end{bmatrix} \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} \begin{bmatrix}\eta_1 \\ \eta_2\end{bmatrix} = \bar\xi_1\,\eta_1 + \bar\xi_2\,\eta_2$$

$$x^1 = \xi^*\,\sigma^1\,\eta = \begin{bmatrix}\bar\xi_1 & \bar\xi_2\end{bmatrix} \begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} \begin{bmatrix}\eta_1 \\ \eta_2\end{bmatrix} = \bar\xi_2\,\eta_1 + \bar\xi_1\,\eta_2$$

$$x^2 = \xi^*\,\sigma^2\,\eta = \begin{bmatrix}\bar\xi_1 & \bar\xi_2\end{bmatrix} \begin{array}{c|c} 0 & -i \\ \hline i & 0 \end{array} \begin{bmatrix}\eta_1 \\ \eta_2\end{bmatrix} = i\,\bar\xi_2\,\eta_1 - i\,\bar\xi_1\,\eta_2$$

$$x^3 = \xi^*\,\sigma^3\,\eta = \begin{bmatrix}\bar\xi_1 & \bar\xi_2\end{bmatrix} \begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array} \begin{bmatrix}\eta_1 \\ \eta_2\end{bmatrix} = \bar\xi_1\,\eta_1 - \bar\xi_2\,\eta_2$$

$$\Rightarrow \text{LHS} = \begin{array}{c|c} x^0-x^3 & -x^1+ix^2 \\ \hline -x^1-ix^2 & x^0+x^3 \end{array} = \begin{array}{c|c} \bar\xi_1\eta_1+\bar\xi_2\eta_2-\bar\xi_1\eta_1+\bar\xi_2\eta_2 & -\bar\xi_2\eta_1-\bar\xi_1\eta_2+i\underbrace{i\bar\xi_2\eta_1-i\bar\xi_1\eta_2} \\ \hline -\bar\xi_2\eta_1-\bar\xi_1\eta_2-i\underbrace{i\bar\xi_2\eta_1-i\bar\xi_1\eta_2} & \bar\xi_1\eta_1+\bar\xi_2\eta_2+\bar\xi_1\eta_1-\bar\xi_2\eta_2 \end{array}$$

$$= 2\begin{array}{c|c} \bar\xi_2\eta_2 & -\bar\xi_2\eta_1 \\ \hline -\bar\xi_1\eta_2 & \bar\xi_1\eta_1 \end{array} = 2\begin{bmatrix}\bar\xi_2 \\ -\bar\xi_1\end{bmatrix}\begin{bmatrix}\eta_2 & -\eta_1\end{bmatrix} = 2\overline{i\sigma^2\xi}\,\overbrace{i\sigma^2\eta}^{T}$$

$$\Phi_R^*\,\sigma^\mu\,\Psi_R = 2\overline{i\sigma^2\Phi_R}\,\overbrace{i\sigma^2\Psi_R}^{T} = 2\,\underbrace{i\sigma^2\bar\Phi_R}\,\overbrace{i\sigma^2\bar\Psi_R}^{*} = 2\,\Phi_L\,\Psi_L^*$$

$$\begin{array}{c|c|c} 0 & x & \Phi_L \\ \hline 0 & 0 & 0 \\ \hline 0 & \Phi_L^* & 0 \end{array} \ast \begin{array}{c|c|c} 0 & y & \Psi_L \\ \hline 0 & 0 & 0 \\ \hline 0 & \Psi_L^* & 0 \end{array} = \begin{array}{c|c|c} 0 & \Phi_L\Psi_L^*+\Psi_L\Phi_L^* & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array} = \begin{array}{c|c|c} 0 & \Phi_R^*\sigma^\mu\Psi_R+\Psi_R^*\sigma^\mu\Phi_R & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}$$

$$\Phi^*_R\,\sigma^\mu\,\Psi_R = 2\overline{i\sigma^2\Phi_R}\,\overbrace{i\sigma^2\Psi_R}^{T} = 2\,\underbrace{i\sigma^2\bar{\Phi}_R}\,\overbrace{i\sigma^2\bar{\Psi}_R}^{*} = 2\,\Phi_L\,\Psi^*_L$$

$$\begin{array}{c|c|c} 0 & x & \Phi_L \\ \hline 0 & 0 & 0 \\ \hline 0 & \Phi^*_L & 0 \end{array} \ast \begin{array}{c|c|c} 0 & y & \Psi_L \\ \hline 0 & 0 & 0 \\ \hline 0 & \Psi^*_L & 0 \end{array} = \begin{array}{c|c|c} 0 & \Phi_L\Psi^*_L + \Psi_L\Phi^*_L & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array} = \begin{array}{c|c|c} 0 & \Phi^*_R\sigma^\mu\Psi_R + \Psi^*_R\sigma^\mu\Phi_R & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}$$

SUSY generators

$$\text{Majorana } Q = Q_a = \begin{bmatrix} Q_L = Q_A \\ Q_R = \bar{Q}^{\dot{A}} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \\ \begin{bmatrix} \bar{Q}^{\dot{1}} \\ \bar{Q}^{\dot{2}} \end{bmatrix} \end{bmatrix}$$

$$-i\,\sigma^2 = \begin{array}{c|c} 0 & -1 \\ \hline 1 & 0 \end{array}$$

$$Q_L = i\,\sigma^2\,\bar{Q}_R$$

$$Q_R = -i\,\sigma^2\,\bar{Q}_L$$

$$\bar{Q}_{\dot{A}} = Q^*_A$$

$$\bar{Q}^{\dot{A}} = \left(-i\sigma^2\right)^{\dot{A}\dot{B}}\,\bar{Q}_{\dot{B}} = \left(-i\sigma^2\right)^{\dot{A}\dot{B}}\,Q^*_B$$

$$\begin{bmatrix} \bar{Q}^{\dot{1}} \\ \bar{Q}^{\dot{2}} \end{bmatrix} = \begin{array}{c|c} 0 & -1 \\ \hline 1 & 0 \end{array} \begin{bmatrix} Q^*_1 \\ Q^*_2 \end{bmatrix} = \begin{bmatrix} -Q^*_2 \\ Q^*_1 \end{bmatrix}$$

$$\begin{bmatrix} Q_L \\ Q_R \end{bmatrix} = \begin{bmatrix} i\sigma^2\bar{Q}_R \\ -i\sigma^2\bar{Q}_L \end{bmatrix} = \underbrace{\begin{array}{c|c} i\sigma^2 & 0 \\ \hline 0 & -i\sigma^2 \end{array}}_{=C} \begin{bmatrix} \bar{Q}_R \\ \bar{Q}_L \end{bmatrix}$$

$$\begin{array}{c|c|c} 0 & x & \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \\ \hline 0 & 0 & 0 \\ \hline 0 & \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}^* & 0 \end{array} \ast \begin{array}{c|c|c} 0 & y & \begin{bmatrix} \widetilde{Q}_1 \\ \widetilde{Q}_2 \end{bmatrix} \\ \hline 0 & 0 & 0 \\ \hline 0 & \begin{bmatrix} \widetilde{Q}_1 \\ \widetilde{Q}_2 \end{bmatrix} L^* & 0 \end{array} = \begin{array}{c|c|c} 0 & \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \begin{bmatrix} \widetilde{Q}_1 \\ \widetilde{Q}_2 \end{bmatrix}^* + \begin{bmatrix} \widetilde{Q}_1 \\ \widetilde{Q}_2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}^* & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}$$

$$= \begin{array}{c|c|c} 0 & \begin{bmatrix} -Q^*_2 \\ Q^*_1 \end{bmatrix}^* \sigma^\mu \begin{bmatrix} -\widetilde{Q}^*_2 \\ \widetilde{Q}^*_1 \end{bmatrix} + \begin{bmatrix} -\widetilde{Q}^*_2 \\ \widetilde{Q}^*_1 \end{bmatrix}^* \sigma^\mu \begin{bmatrix} -Q^*_2 \\ Q^*_1 \end{bmatrix} & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}$$

$$
= \left[\begin{array}{c|c|c}
0 & \begin{bmatrix} -Q_2 & Q_1 \end{bmatrix} \sigma^\mu \begin{bmatrix} -\widetilde{Q}^*_2 \\ \widetilde{Q}^*_1 \end{bmatrix} + \begin{bmatrix} -\widetilde{Q}_2 & \widetilde{Q}_1 \end{bmatrix} \sigma^\mu \begin{bmatrix} -Q^*_2 \\ Q^*_1 \end{bmatrix} & 0 \\
\hline
0 & 0 & 0 \\
\hline
0 & 0 & 0
\end{array}\right.
$$