

$${}^x \boxed{\mathfrak{N}:\mathfrak{N}} = {}_{\mu} \mathfrak{N} \eta^{\mu\nu} {}_{\nu} \mathfrak{N} - \mathcal{V}_{\mathfrak{N}}$$

$$\tilde{\mathcal{L}}_{\tilde{\mathfrak{N}}:\tilde{\mathfrak{N}}} \stackrel{\text{inv}}{=} {}^x \mathfrak{L}^{\mathfrak{L}+\mathfrak{L}} \mathcal{L}_{\mathfrak{N}:\bar{\mathfrak{L}}\mathfrak{N}} = {}^x \mathcal{L}_{\mathfrak{N}:\mathfrak{N}}$$

$$\det {}^x \mathfrak{L} = \det \mathfrak{L} = 1$$

$$\mathfrak{L} \eta \mathfrak{L}^T = \eta \Rightarrow \bar{\mathfrak{L}}^T \bar{\eta}^1 \bar{\mathfrak{L}} = \bar{\eta}^1 \Rightarrow {}_x \bar{\mathfrak{L}}^{\mu} \eta^{\lambda\nu} {}_{\lambda} \bar{\mathfrak{L}}^{\nu} = \eta^{\mu\nu}$$

$$\underbrace{{}_x \bar{\mathfrak{L}}^{\mu} {}_{\mu} \mathfrak{N} \eta^{\lambda\nu} {}_{\lambda} \bar{\mathfrak{L}}^{\nu} {}_{\nu} \mathfrak{N}} = {}_{\mu} \mathfrak{N} \underbrace{{}_x \bar{\mathfrak{L}}^{\mu} \eta^{\lambda\nu} {}_{\lambda} \bar{\mathfrak{L}}^{\nu}}_{= \eta^{\mu\nu}} {}_{\nu} \mathfrak{N} = {}_{\mu} \mathfrak{N} \eta^{\mu\nu} {}_{\nu} \mathfrak{N}$$

$$\tilde{\mathcal{L}}_{\tilde{\mathfrak{N}}:\tilde{\mathfrak{N}}} = {}^x \mathfrak{L}^{\mathfrak{L}+\mathfrak{L}} \mathcal{L}_{\mathfrak{N}:\bar{\mathfrak{L}}\mathfrak{N}} = \underbrace{{}_x \bar{\mathfrak{L}}^{\mu} {}_{\mu} \mathfrak{N} \eta^{\lambda\nu} {}_{\lambda} \bar{\mathfrak{L}}^{\nu} {}_{\nu} \mathfrak{N}} - \mathcal{V}_{\mathfrak{N}} = {}_{\mu} \mathfrak{N} \eta^{\mu\nu} {}_{\nu} \mathfrak{N} - \mathcal{V}_{\mathfrak{N}} = {}^x \mathcal{L}_{\mathfrak{N}:\mathfrak{N}}$$